ICAP Summer School 2016

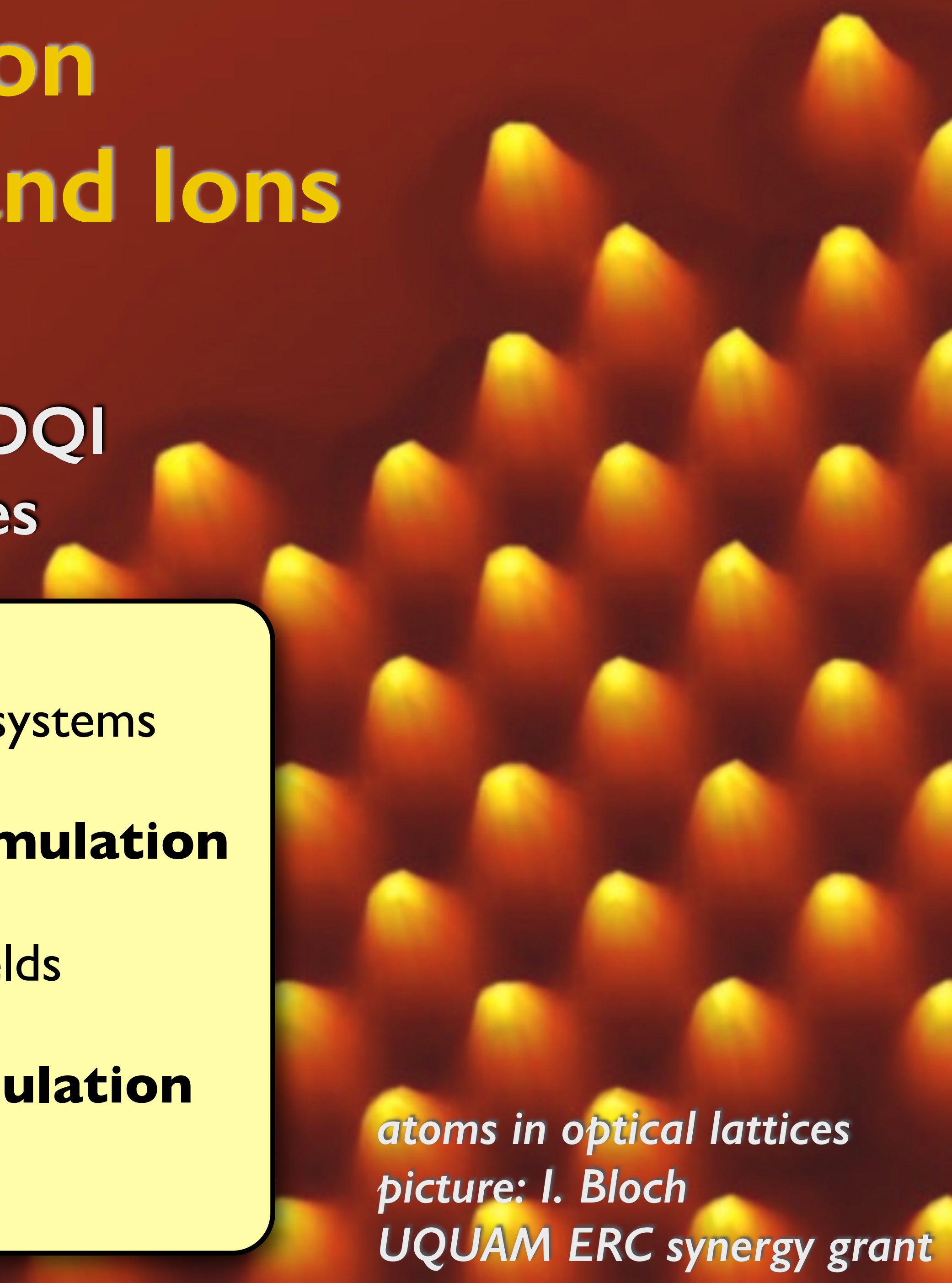
Quantum Simulation with Cold Atoms and Ions

Peter Zoller
University of Innbruck & IQOQI
Austrian Academy of Sciences

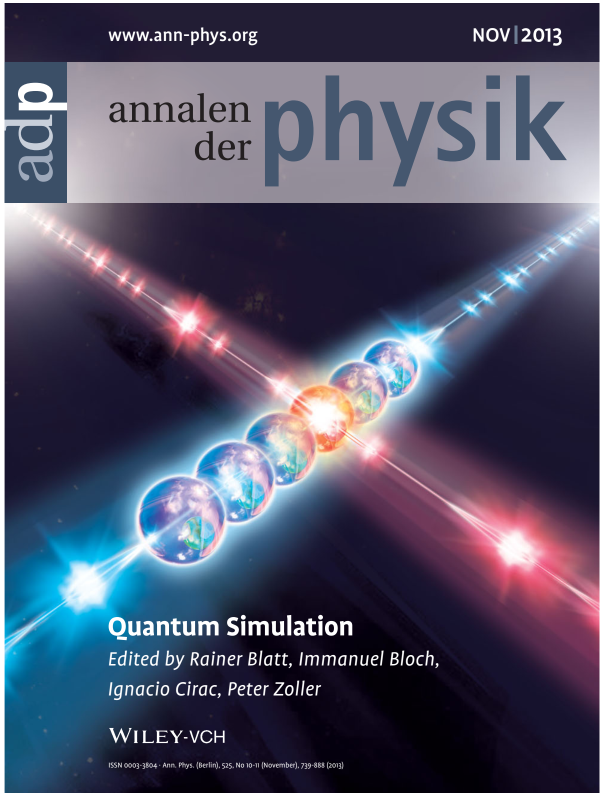
Overview / Introduction
- From concepts to quantum optical systems

Lecture 1: 'Closed System' QSimulation
- Measuring 'Entanglement'
- From Static to Dynamical Gauge Fields

Lecture 2: 'Open System' QSimulation
- Quantum Reservoir Engineering
- 'Chiral Quantum Optics'



atoms in optical lattices
picture: I. Bloch
UQUAM ERC synergy grant



Introduction & Overview:

Quantum Simulation with Atoms, Molecules & Ions

Nature Physics Insight Surveys (2012)
Quantum Simulations

- J. I. Cirac and P. Zoller, *Goals and Opportunities in Quantum Simulation*
- I. Bloch, J. Dalibard, and S. Nascimbene, *QS with Ultracold Quantum Gases*
- R. Blatt and C. F. Roos, *QS with Trapped Ions*
- A. Aspuru-Guzik and P. Walter, *Photonic QS*

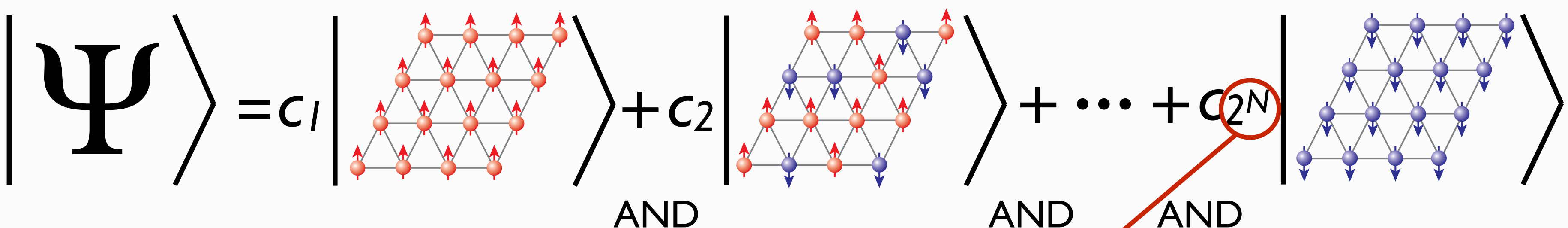
'The Credo' of Quantum Simulation

Schrödinger

- **QUANTUM many-body systems:**

  The system is in a *superposition* state of all possible configurations …

Entanglement



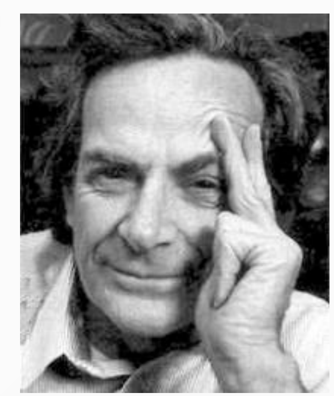
Exponentially large Hilbert space

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

a thing which is usually described by local differential equations. But the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics—which is what I really want to talk about, but I'll come to that later. So what kind of simulation do I mean?



Experimental Quantum Simulation

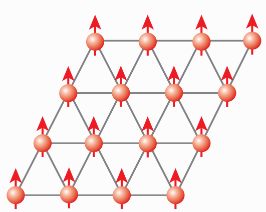
- **QUANTUM many-body systems:**

  The system is in a *superposition* state of all possible configurations …

Schrödinger

Entanglement

$$|\Psi\rangle = c_1 |\cdots\rangle + c_2 |\cdots\rangle + \cdots + c_{2^N} |\cdots\rangle$$



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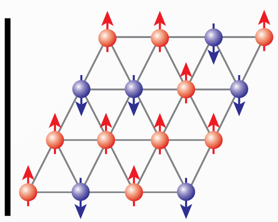
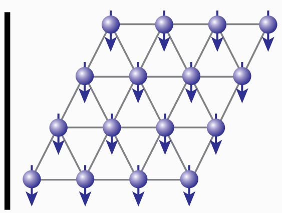


AND AND



- **Building Quantum Simulators with AMO-systems, [solid state] …**
  - cold atoms in optical lattices
  - trapped ions
  - photons …

controlled many-body quantum systems
- dynamics: closed / open
- preparation & measurement

Can we measure Entanglement?

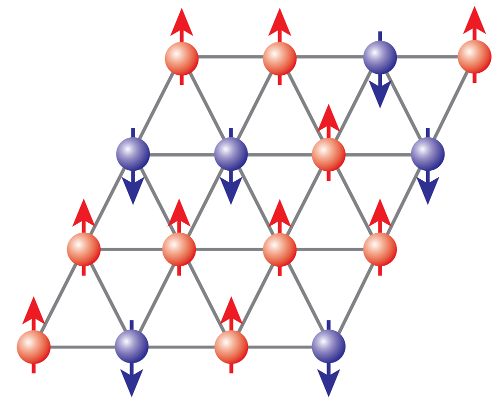
UQUAM
ERC Synergy Grant

Entanglement & Quantum Devices

- **QUANTUM physics:**

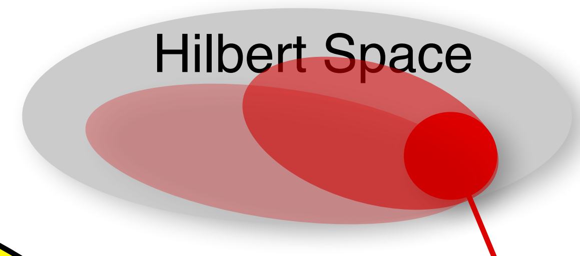
  The system is in a *superposition* state of all possible configurations …

  “Entanglement”



$$|\Psi\rangle = c_1|\uparrow\uparrow\uparrow\ldots\uparrow\uparrow\rangle + c_2|\uparrow\uparrow\uparrow\ldots\uparrow\downarrow\rangle + \ldots + c_{2^N}|\downarrow\downarrow\downarrow\ldots\downarrow\downarrow\rangle$$

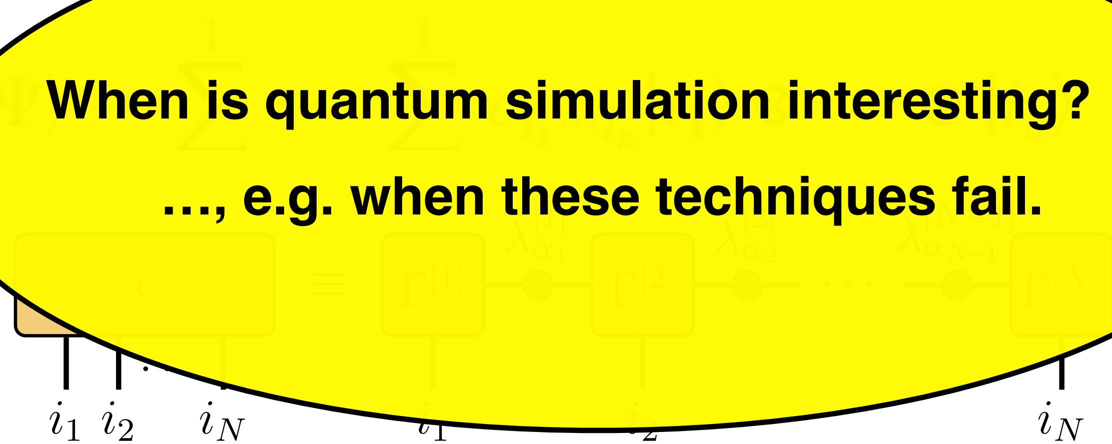
- **Remark: Quantum Information provides insight into *classical* simulations**



When is quantum simulation interesting?

…, e.g. when these techniques fail.

MATRIX PRODUCT STATE



Replaced the 2^N coefficients with ~ $(2\chi^2 + \chi)$ n coefficients

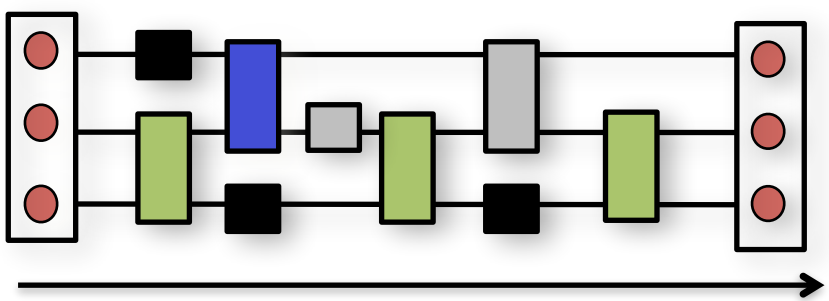
- 1D: tDMRG
- 2D: PEPS, MERA

- eigenstates
- time dependent

Quantum Info

- **general purpose quantum computing**

quantum logic network model

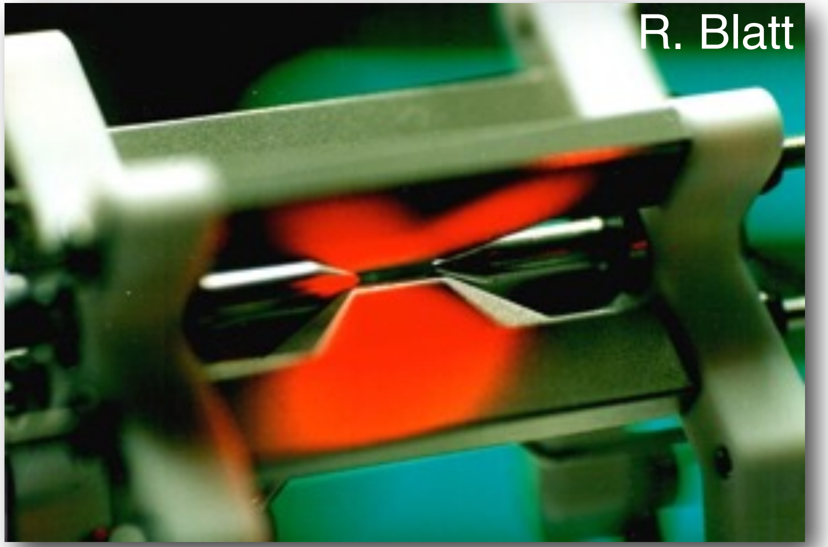


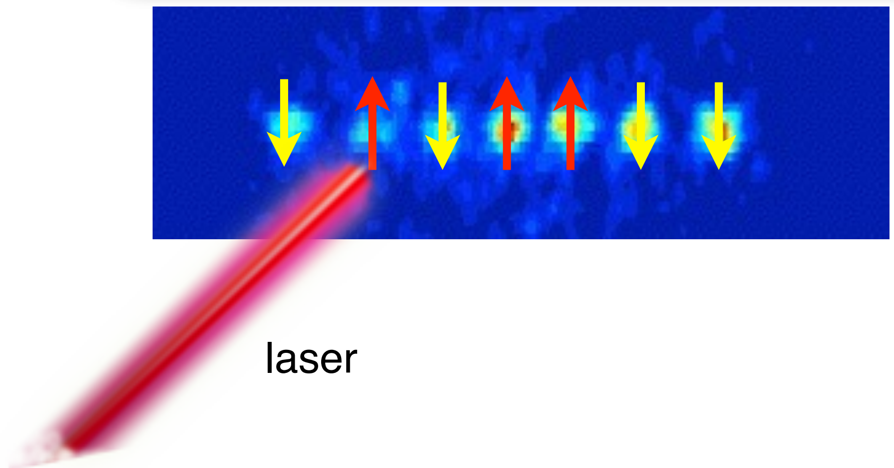
$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

coherent Hamiltonian evolution
- quantum gates
- deterministic

Quantum Optics

- **atomic physics: trapped ions**





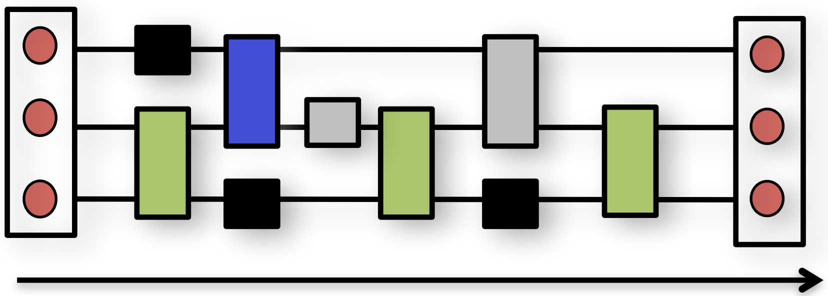
Exp.: Innsbruck, NIST, JQI, MIT, Mainz, MPQ ...

Quantum Info

- **general purpose quantum computing**

quantum logic network model

qubits quantum gates read out



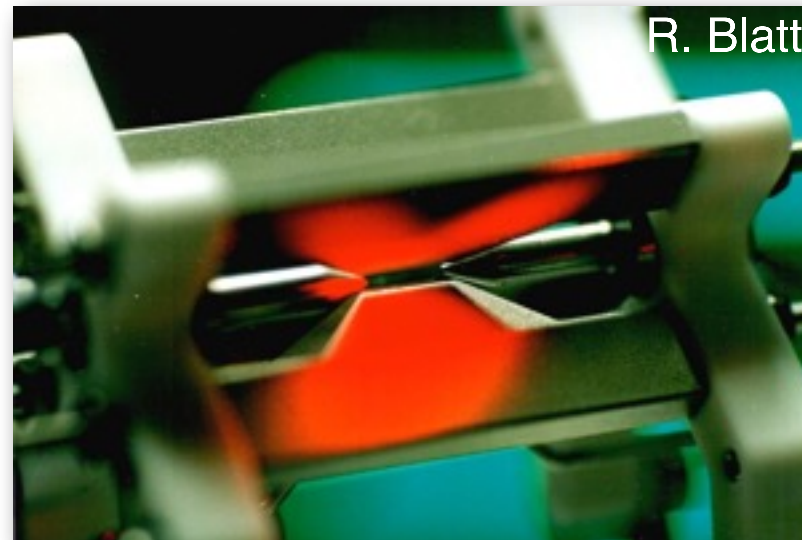
time

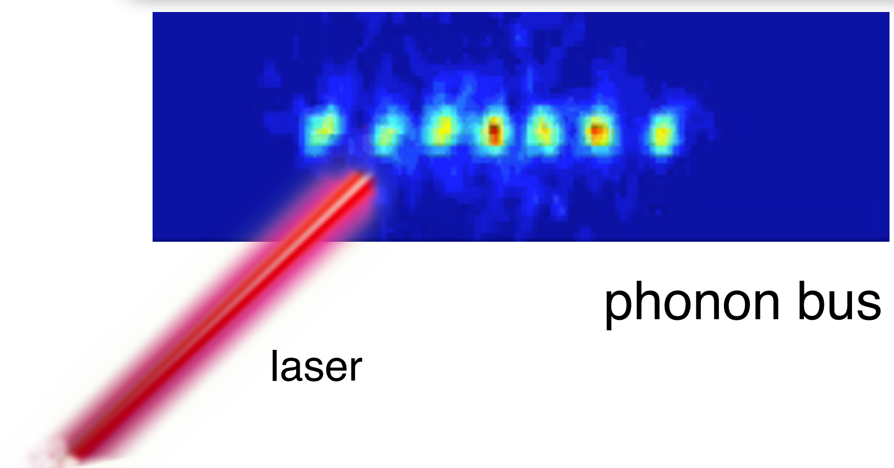
$$|\psi\rangle \rightarrow U_t\,|\psi\rangle$$

coherent Hamiltonian evolution
- quantum gates
- deterministic

Quantum Optics

- **atomic physics: trapped ions**





phonon bus

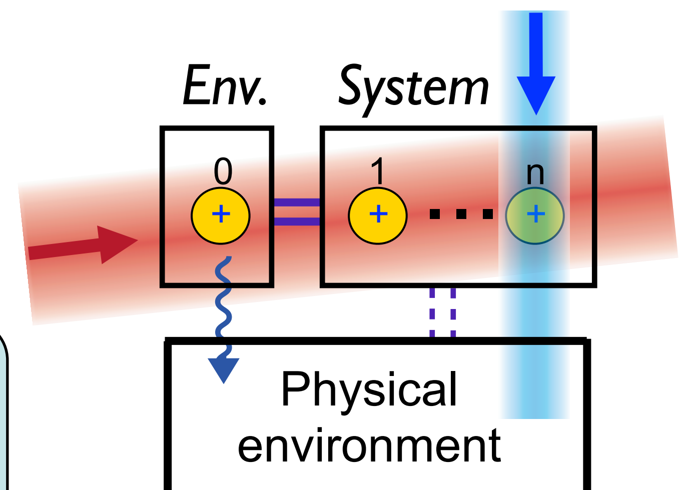
laser

Exp.: Innsbruck, NIST, JQI, MIT, Mainz, MPQ ...

Quantum operations on Innsbruck ion-trap quantum computer

Individual light-shift gates

$\sigma_z^{(0)}, \sigma_z^{(1)}, \sigma_z^{(2)}$



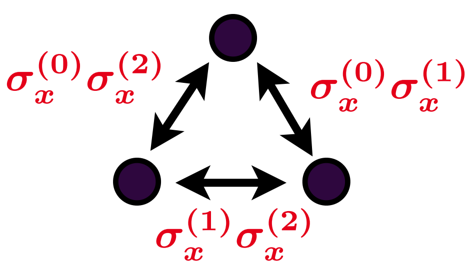
Collective spin flips

S_x, S_y

Mølmer-Sørensen gate

$$S_x^2 = \sigma_x^{(0)}\sigma_x^{(1)} + \sigma_x^{(1)}\sigma_x^{(2)} + \sigma_x^{(0)}\sigma_x^{(2)}$$

Coupling of environment with physical environment

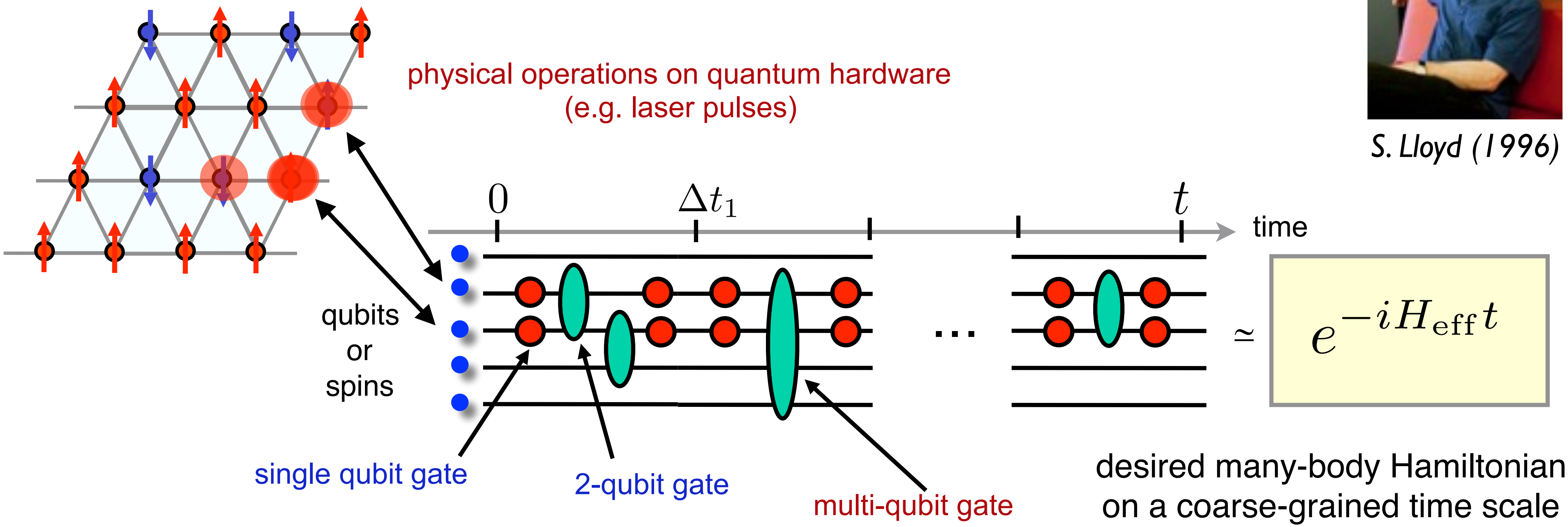


Optical pumping of „environmental“ ion

J. Barreiro, M.Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller and R. Blatt, Nature 2011

Digital Quantum Simulation

S. Lloyd (1996)



idea: approximate time evolution by a stroboscopic sequence of gates

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots^{-iH\Delta t_1/\hbar}$$

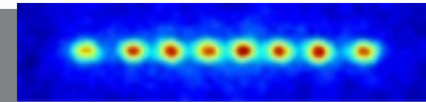
Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \; \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \; \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\text{Trotter errors for non-commuting terms}}$$

$$H = \boxed{-J\sigma_1^z\sigma_2^z} + \boxed{B(\sigma_1^x + \sigma_2^x)}$$

... not restricted to unitary dynamics

SCIENCE VOL 334 7 OCTOBER 2011

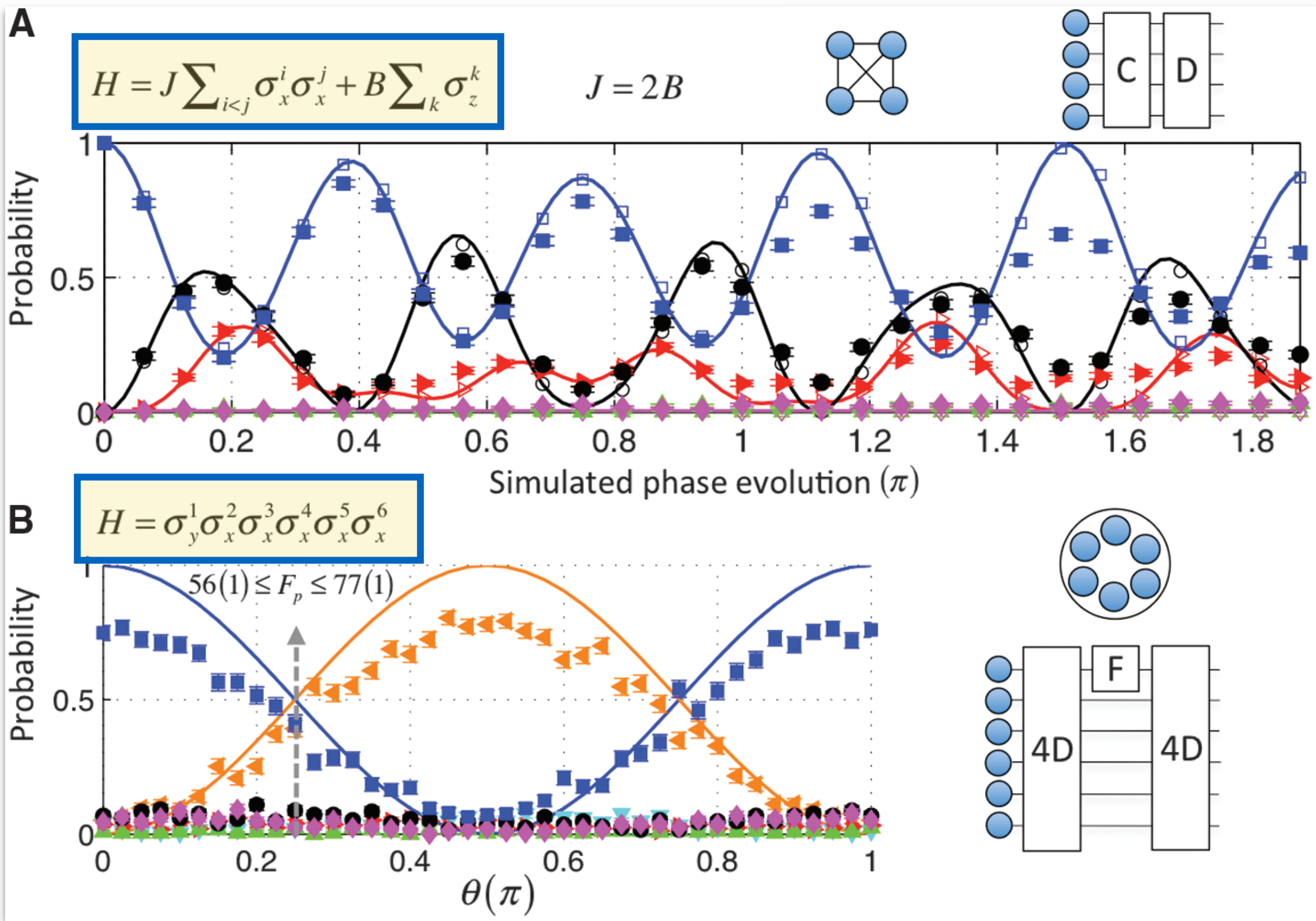


REPORTS

Universal Digital Quantum Simulation with Trapped Ions

B. P. Lanyon,[1,2*] C. Hempel,[1,2] D. Nigg,[2] M. Müller,[1,3] R. Gerritsma,[1,2] F. Zähringer,[1,2] P. Schindler,[2] J. T. Barreiro,[2] M. Rambach,[1,2] G. Kirchmair,[1,2] M. Hennrich,[2] P. Zoller,[1,3] R. Blatt,[1,2] C. F. Roos[1,2]

four and six spin systems



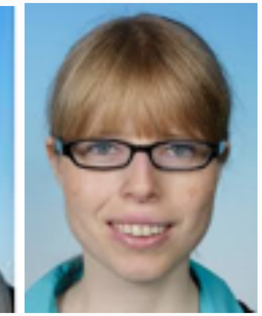
remarks:

- scalability (?)
- error correction (?)

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

doi:10.1038/nature18318

E. Martinez C. Muschik

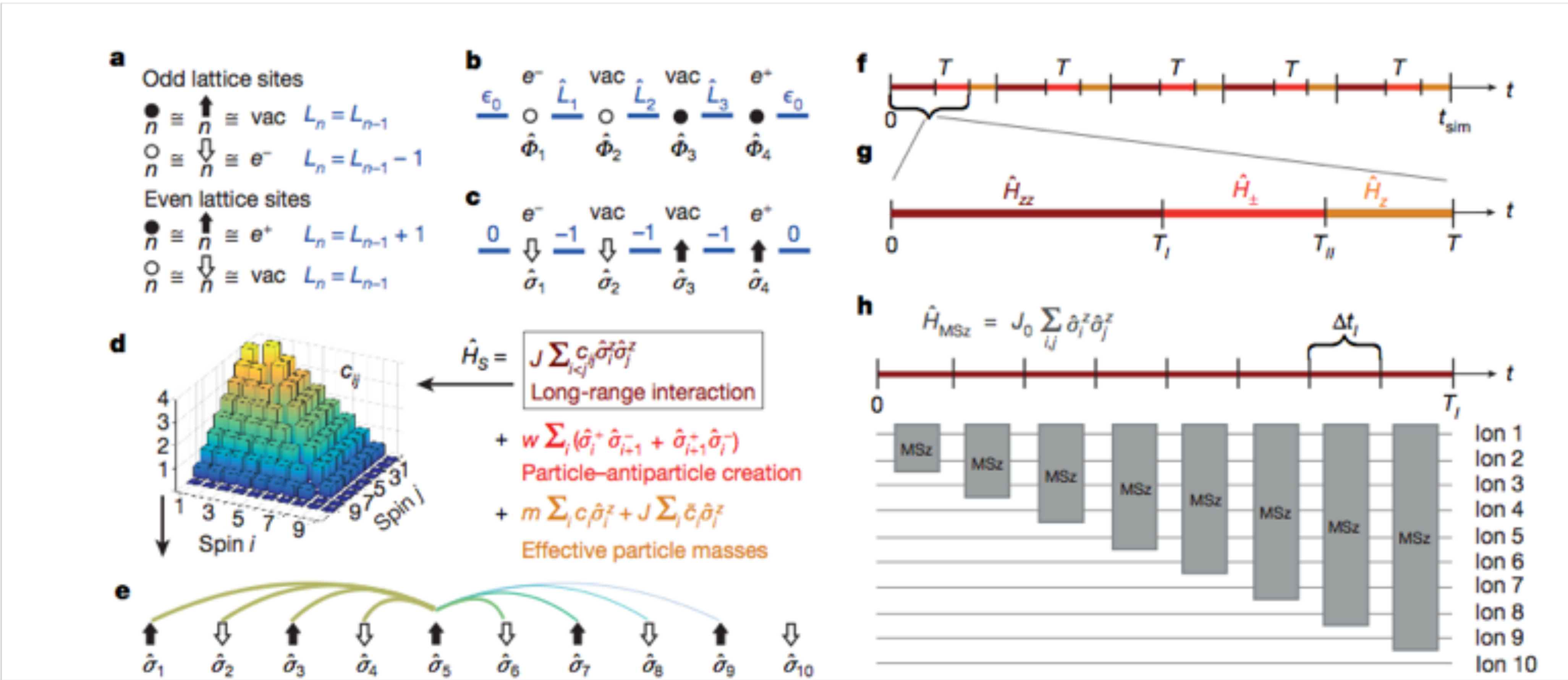


Figure 2 | Encoding Wilson's lattice gauge theories in digital quantum simulators. Matter fields, represented by one-component fermion fields

see talk by R. Blatt this Saturday

Quantum Info

Quantum Optics

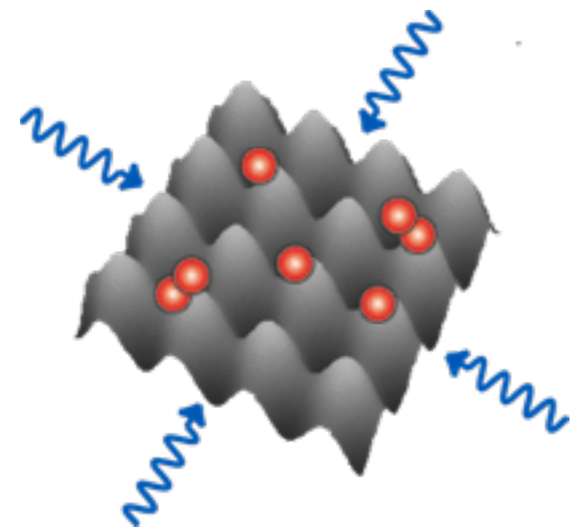
- **Hubbard models etc.**

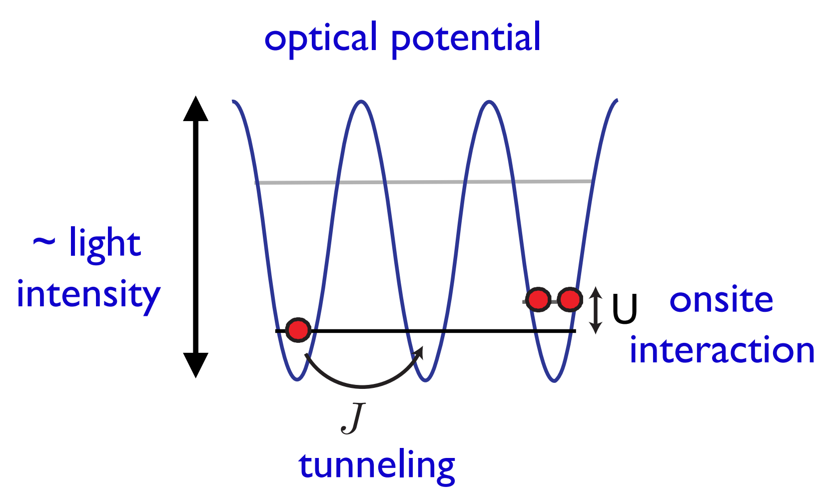
$$\hat{H} = -\sum_{\alpha \neq \beta} J_{\alpha\beta} \hat{a}_\alpha^\dagger a_\beta + \frac{1}{2} U \sum_\alpha \hat{a}_\alpha^\dagger \hat{a}_\alpha^\dagger \hat{a}_\alpha \hat{a}_\alpha$$

Hubbard Hamiltonian

Bosons, Fermions
- strongly correlated system
- quantum phase transitions

- **atoms or molecules in optical lattices [and ions]**





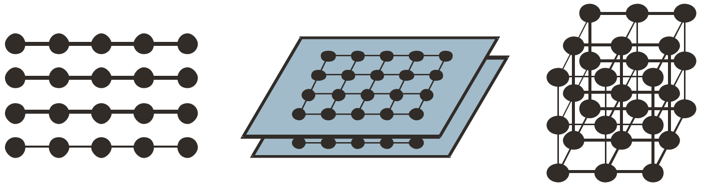
Analog quantum simulation: “always on”

- We “build” a quantum system with desired Hamiltonian & *controllable parameters*, e.g. Hubbard models of atoms in optical lattices

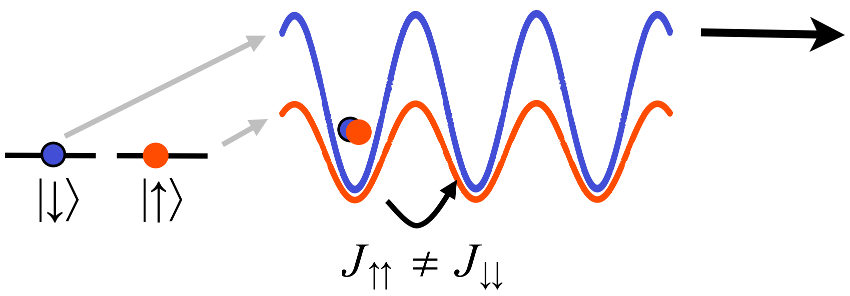
Hubbard Toolbox

D. Jaksch & PZ,
Annals of Physics 2005

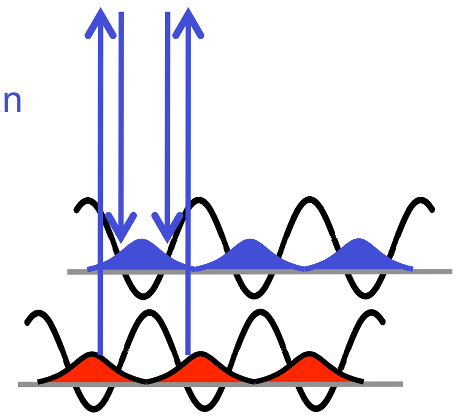
- time dependence, 1D, 2D & 3D
- various lattice configurations



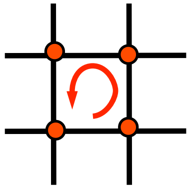
- spin-dependent lattices



- laser induced hoppings

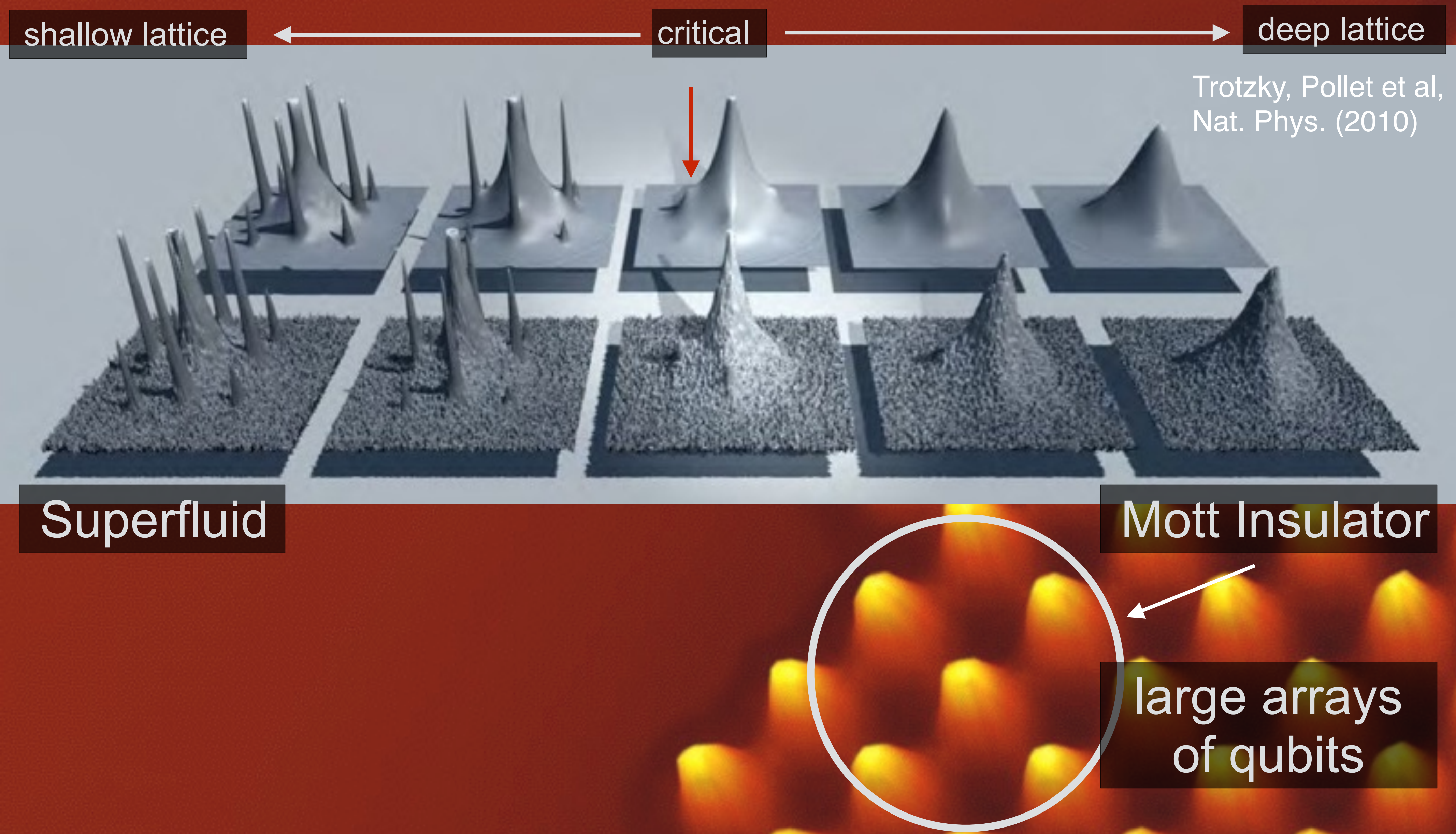


- create effective magnetic fields



$$J_{\alpha\beta} \longrightarrow J_{\alpha\beta}\, e^{ie\int_\alpha^\beta \vec{A}\cdot d\vec{l}}$$

Superfluid - Mott Insulator Quantum Phase Transition

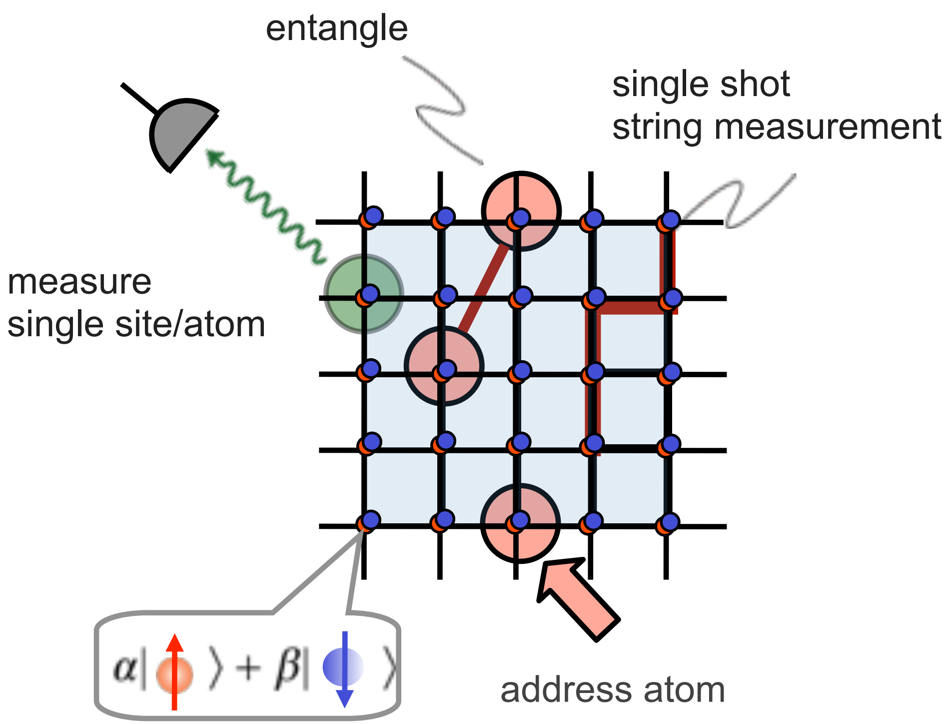


Quantum Info

Quantum Optics

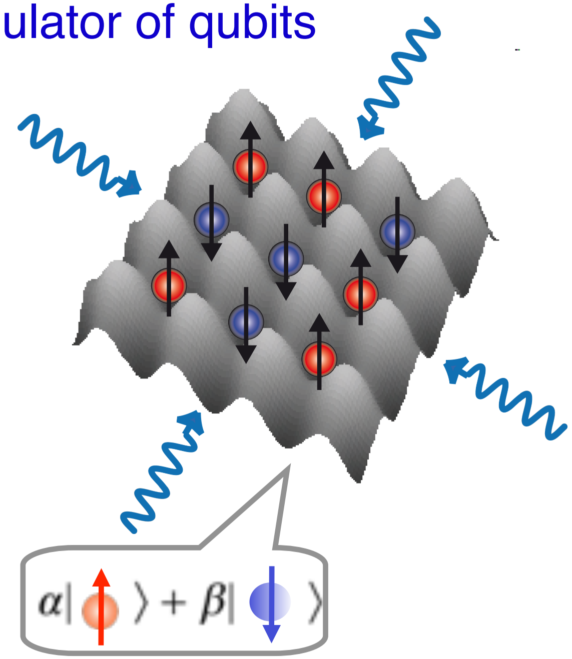
- **general purpose quantum computing**

quantum logic network model



- **atoms or molecules in optical lattices [and ions]**

Mott insulator of qubits



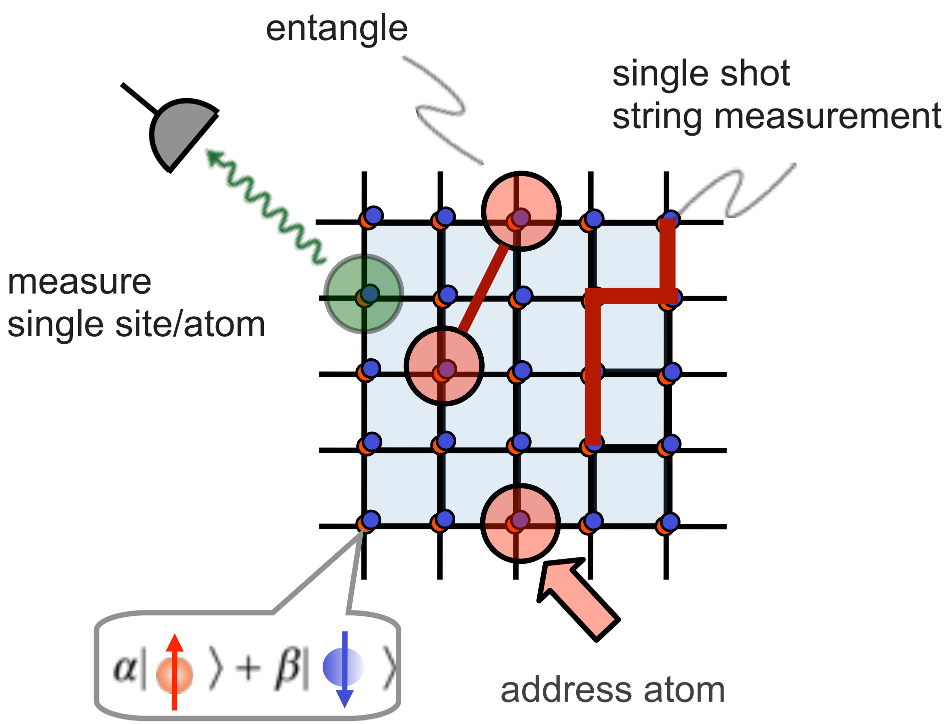
filling the lattice with “qubits”

a bottom up approach

Quantum Info

- **general purpose quantum computing**

quantum logic network model



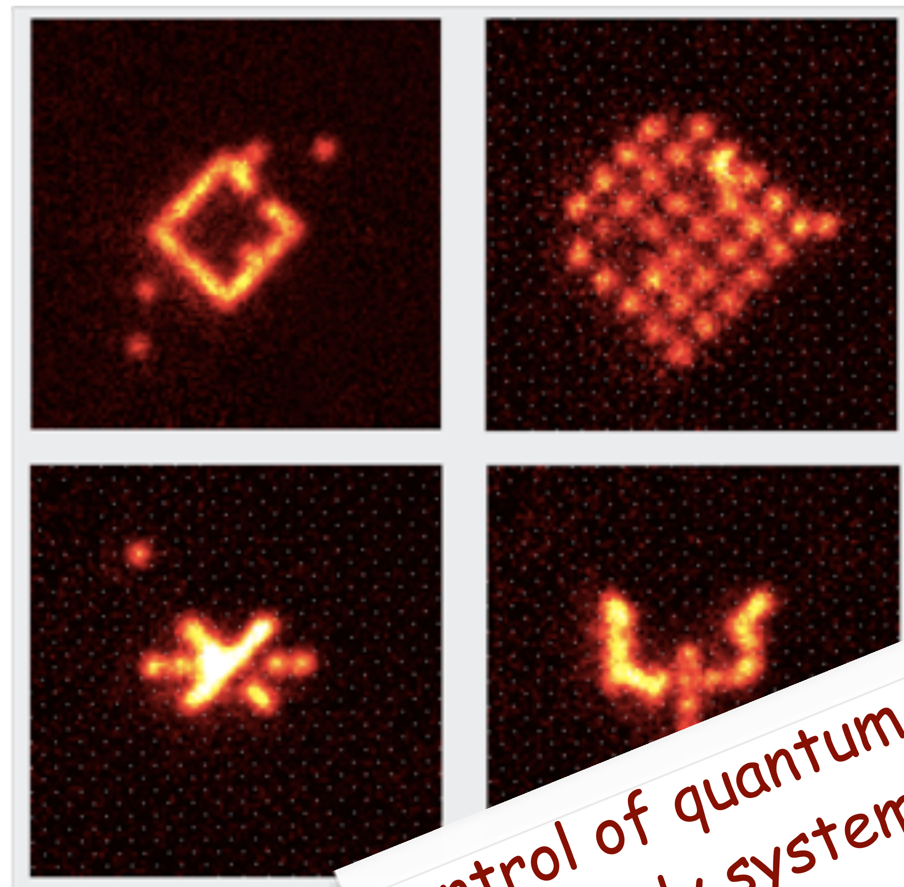
entangling gates / interactions

✓ Rydberg, collisions, CQED

Quantum Optics

- **single site addressing**

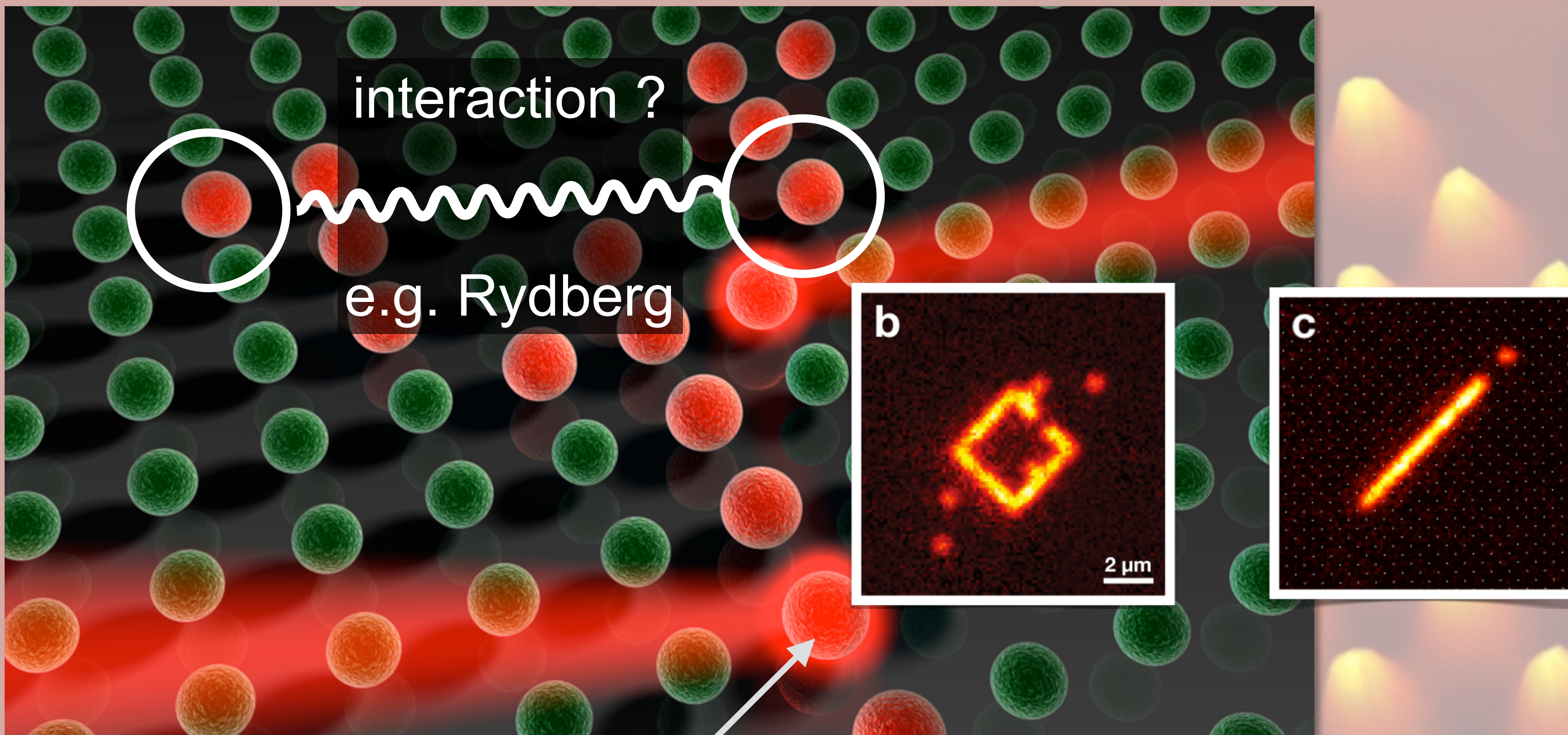
Harvard, MPQ, Chicago
large spacing lattices: Paris, Madison, Penn State, …



Single-Spin Addr… …or,
I Bloch, S Kuhr et …

control of quantum
many-body system on
level of single quanta

Control of Atoms in Optical Lattices



single site addressing


REPORTS

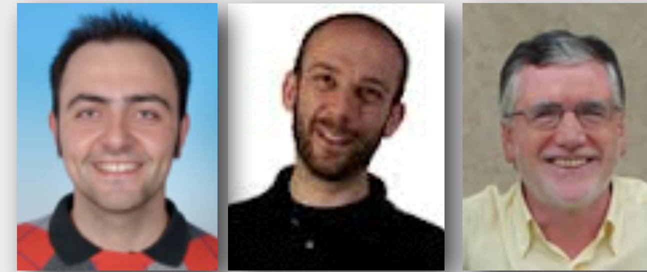
SCIENCE

25 SEPTEMBER 2015 • VOL 349 ISSUE 6255

QUANTUM SIMULATION

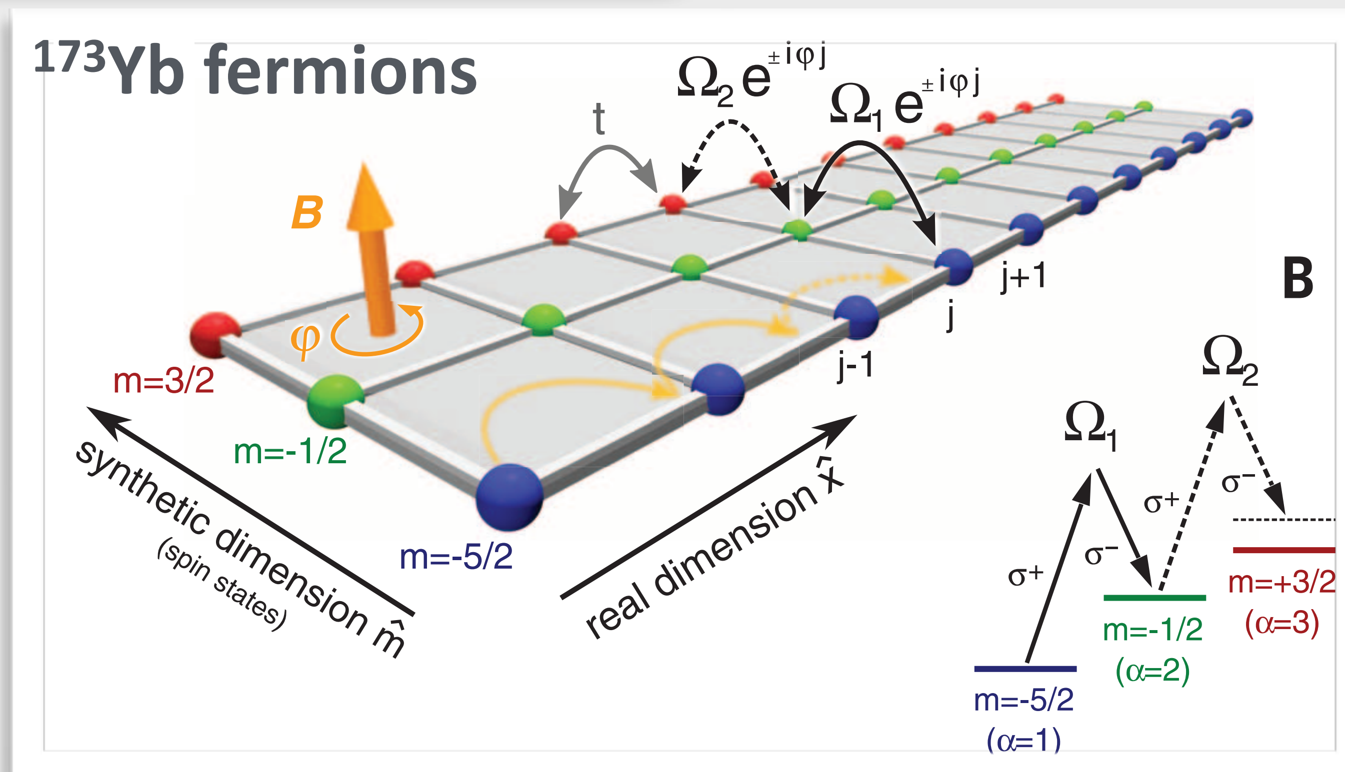
Observation of chiral edge states with neutral fermions in synthetic Hall ribbons

M. Mancini,[1] G. Pagano,[1] G. Cappellini,[2] L. Livi,[2] M. Rider,[3,4] J. Catani,[5,2] C. Sias,[6,2] P. Zoller,[3,4] M. Inguscio,[6,1,2] M. Dalmonte,[3,4] L. Fallani[1,2]*



M. Dalmonte | L. Fallani | M. Inguscio

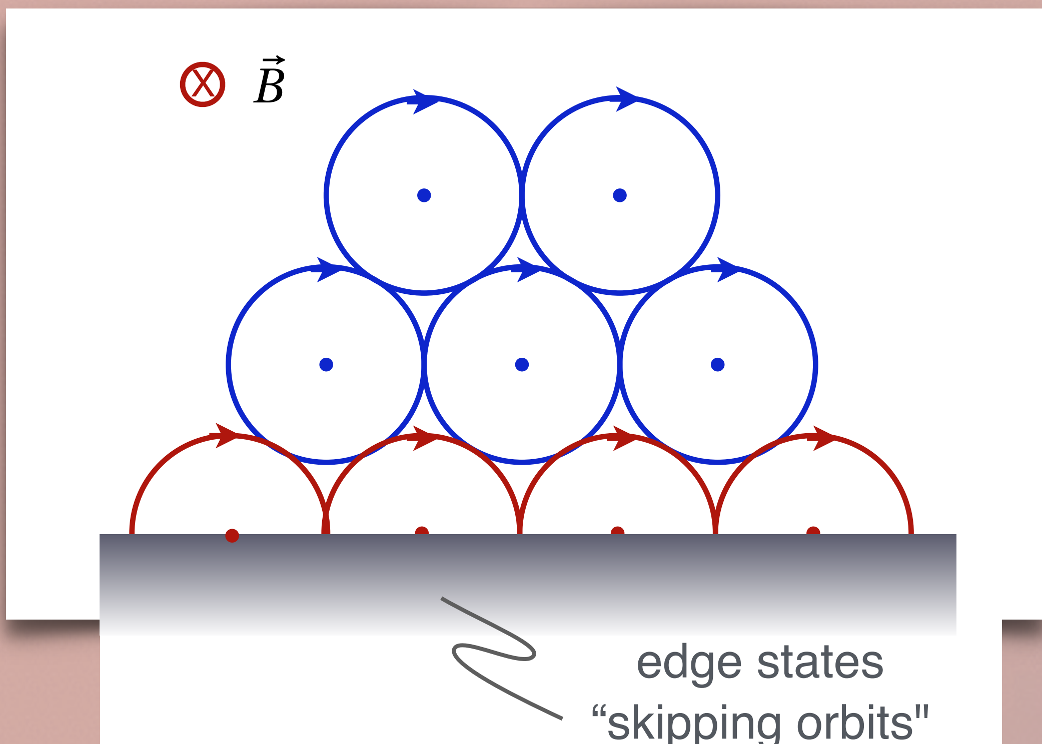
UIBK | LENS, Florence



See also: I. Spielman et al., *ibid.* (bosons)

Particle in a [synthetic] magnetic field

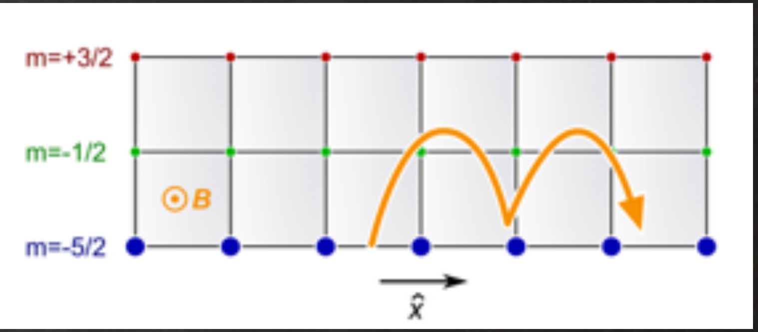
cyclotron orbit in B-field



quantum wave packet

*M. Mancini et al., Science **349**, 1510 (2015)*

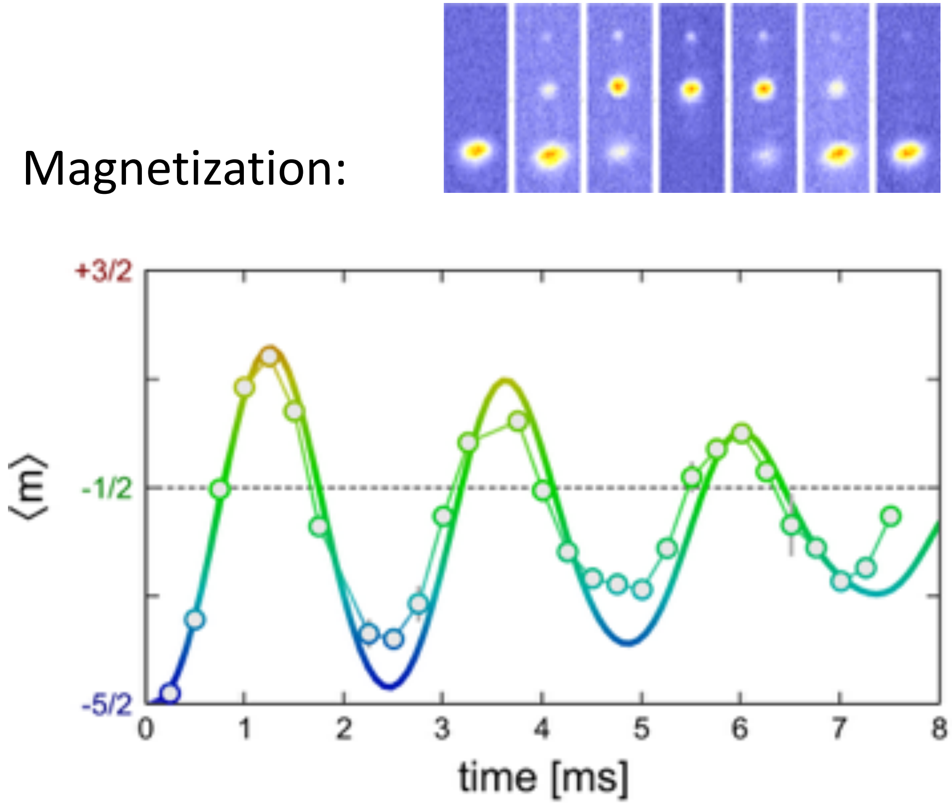
Edge-cyclotron orbits



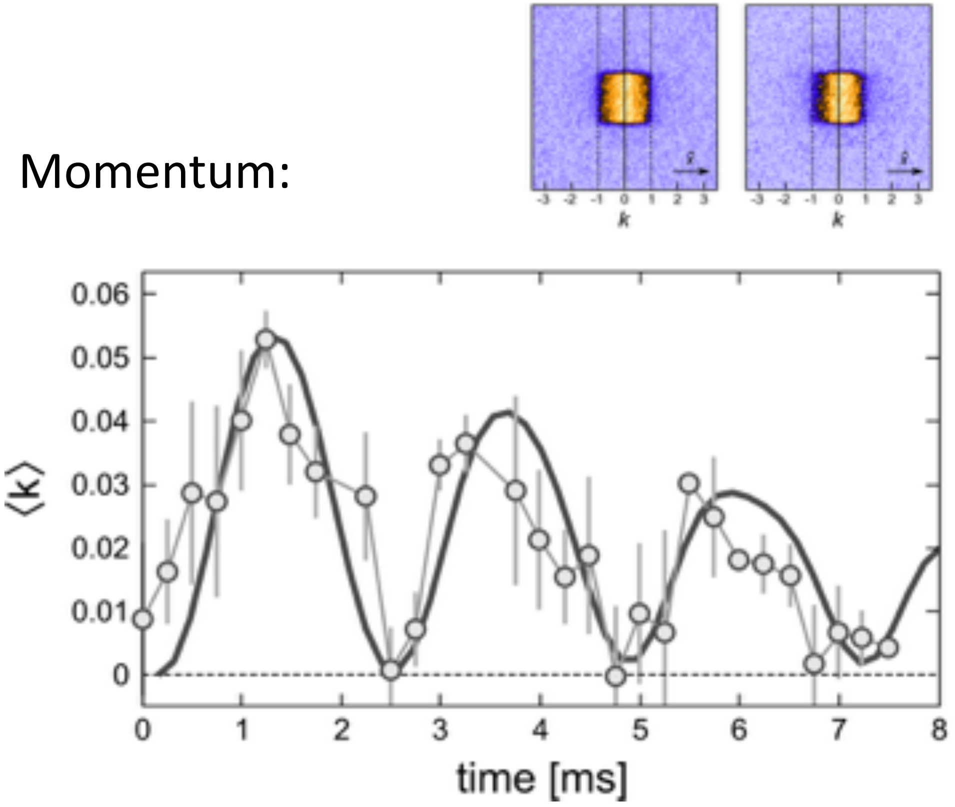
Initial state with <k>=0 on the m=-5/2 leg

Quenched dynamics after activation of synthetic tunneling

Magnetization:



Momentum:

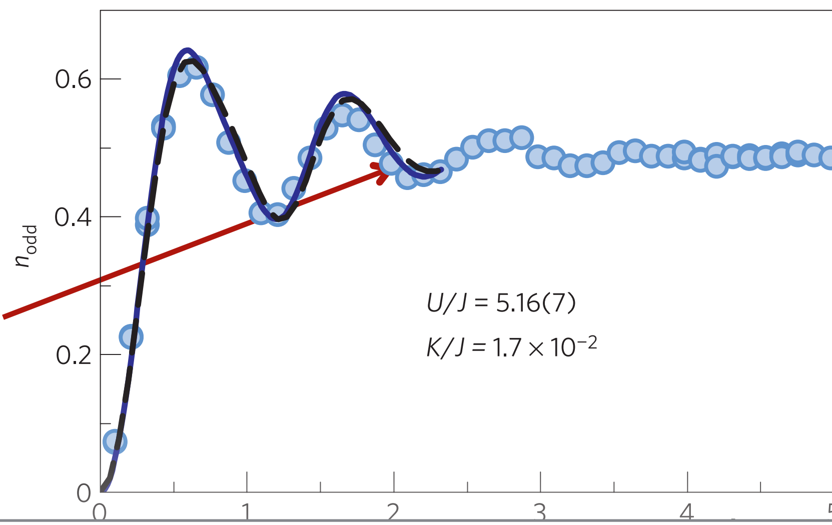


nature physics
ARTICLES
PUBLISHED ONLINE: 19 FEBRUARY 2012 | DOI: 10.1038/NPHYS2232

Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas

S. Trotzky[1,2,3]★, Y-A. Chen[1,2,3], A. Flesch[4]★, I. P. McCulloch[5], U. Schollwöck[1,6], J. Eisert[6,7,8] and I. Bloch[1,2,3]

tDMRG (theory)
Breaks down here



Can we measure entanglement growth (& purity)?

☑ Efficiently (?)
Mott insulator
BEC
☑ With the available tools in cold atom experiments

A. J. Daley, HP, J. Schachenmayer, P. Zoller PRL 2012; NJP 2013

entanglement growth: A.M. Läuchli and C. Kollath, J. Stat. Mech. 2008, P05018 (2008).

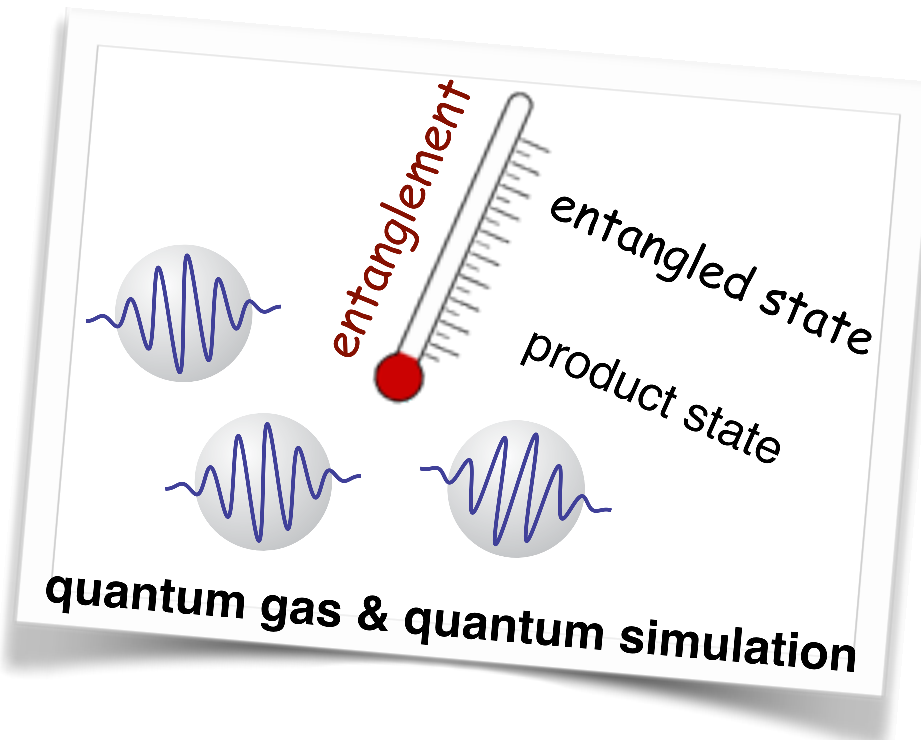
Special Topic 1:

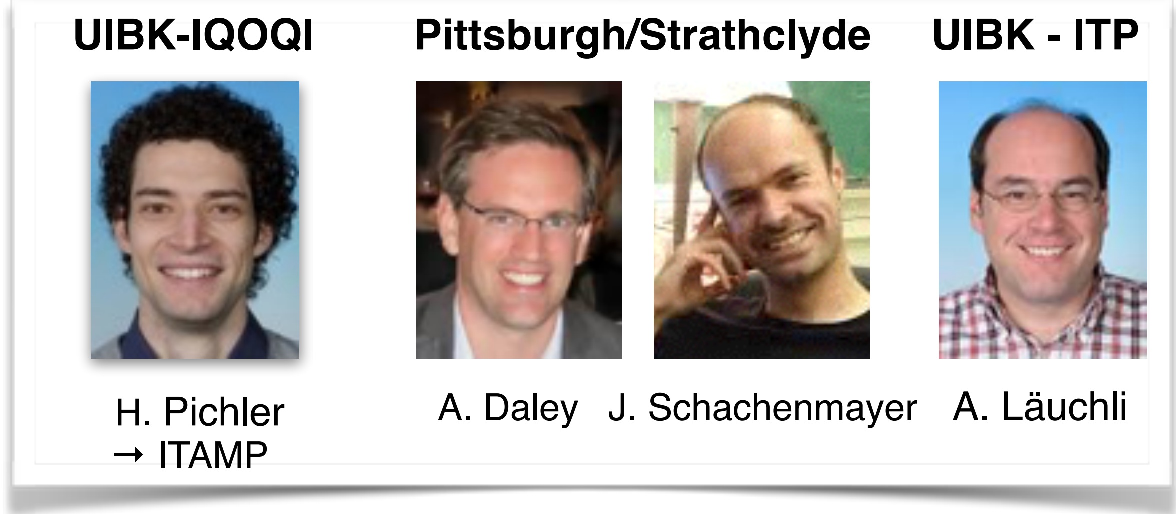
Measuring Entanglement

Cold Atoms in Optical Lattices

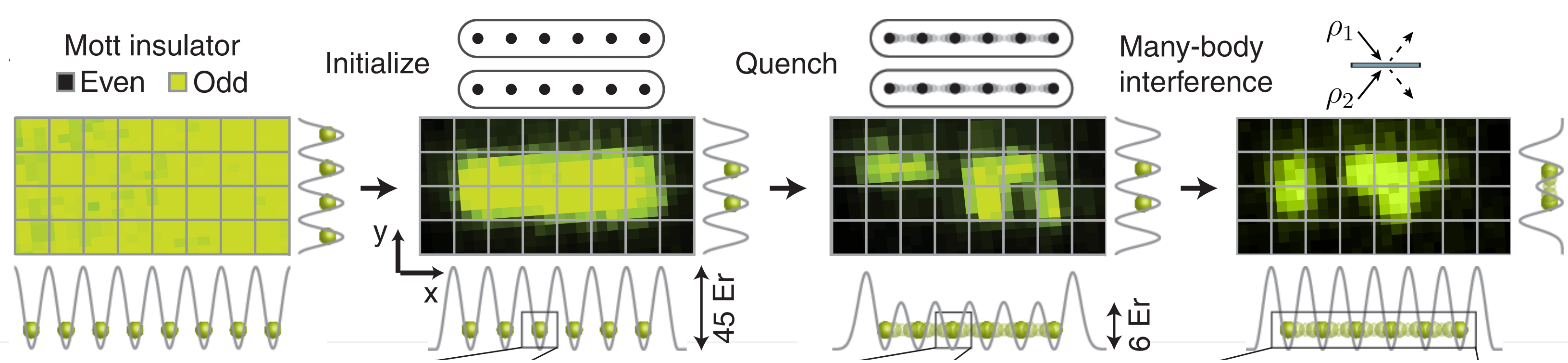
… developing new measurement protocols in AMO

… based on new tools:
quantum gas microscope





Controlled few-atom systems & quantum gas microscope

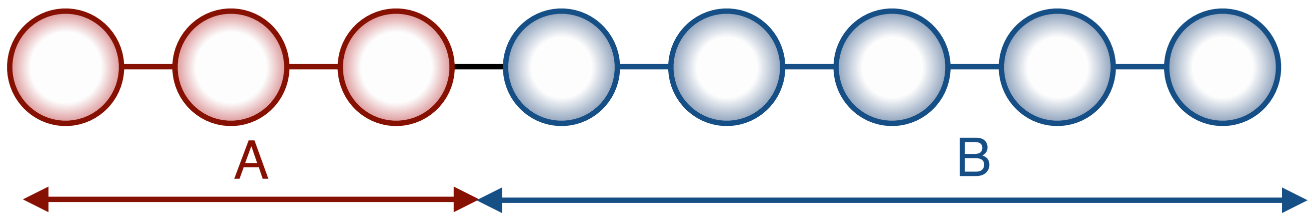


- experiment

R. Islam, M. Greiner et al., Nature (2015)

A.M. Kaufmann, M. Greiner et al., arXiv 2016

Entanglement Measures



- **For pure state of the total system**

Product state of A and B

$$|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$$

reduced density matrix

$$\rho_A = \mathrm{tr}_B\{\rho\} = |\Psi_A\rangle \otimes \langle\Psi_A|$$

Entangled state of A and B

$$|\Psi\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$$

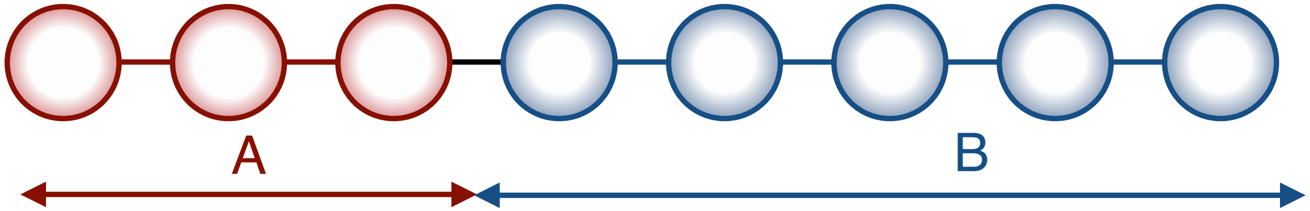
- **Measurement of bipartite entanglement**

$$S_{VN}(\rho_A) = -\mathrm{tr}\{\rho_A \log \rho_A\} = 0$$

$$S_{VN}(\rho_A) > 0$$

Reviews: L. Amico, R. Fazio, A. Osterloh and V. Vedral, RMP (2008)
J. Eisert, M. Cramer and M. B. Plenio, RMP (2010)
P. Calabrese, J. Cardy and B. Doyon, JPA (2009)
I. Peschel and V. Eisler, JPA (2009)
O. Gühne, and G. Tóth, Phys. Rep. (2009).

Entanglement Measures



- Another measure is the **Rényi entropy** of order n, which bounds the von Neumann entropy

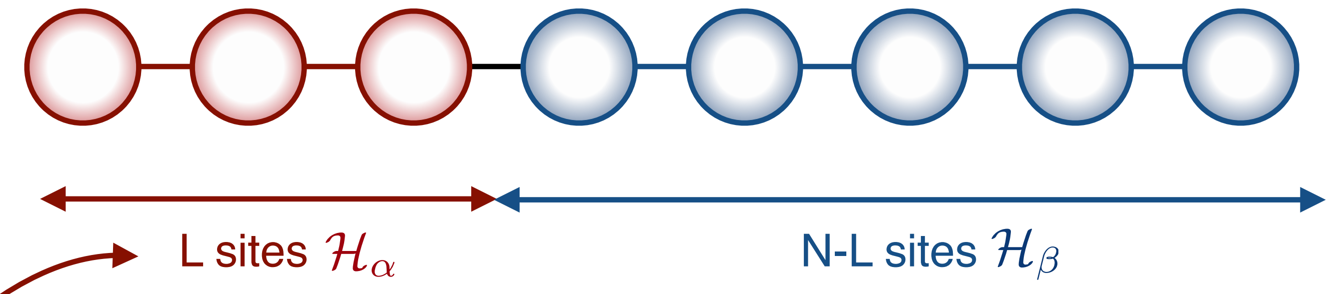
$$S_n(\rho_A) = \frac{1}{1-n} \log \mathrm{tr}\{\rho_A^n\} \leq S_{VN}(\rho_A)$$

and also measures the concurrence

Properties:

- $S_{VN}(\rho) \; [= \lim_{n \to 1} S_n(\rho)]$
- $S_{VN}(\rho) \geq S_2(\rho)$
- $S_{VN}(\rho) \geq 2S_2(\rho) - S_3(\rho)$

Measuring Rényi Entropies … the Challenge



Reduced state $\rho_\alpha = \mathrm{Tr}_{\mathcal{H}_\beta}\{\rho\}$ $= \sum_k \lambda_k |\phi_k\rangle\langle\phi_k|$

entanglement spectrum

- **Rényi entropy of order n**

$$S_n(\rho_\alpha) \equiv \frac{1}{1-n} \log \mathrm{tr}\{\rho_\alpha^n\}$$

Can we measure $\mathrm{Tr}\{\rho_\alpha^n\}$?

- **Mixed** states: inequalities bounds

$$S_n(\rho) < S_n(\rho_\alpha) \rightarrow E(\rho) > 0$$

$$\sqrt{2(\mathrm{Tr}\{\rho^2\} - \mathrm{Tr}\{\rho_\alpha^2\})} \leq c(\rho) \leq \sqrt{2(1 - \mathrm{Tr}\{\rho_\alpha^2\})}$$

F. Mintert et al., Phys. Rev. Lett. 95, 260502 (2005).

Protocols to measure Rényi entropies

- **Quantum Tomography of the density matrix**

  however, …

Quantum Tomography to measure ρ

Vol 438 | 1 December 2005 | doi:10.1038/nature04279

Scalable multiparticle entanglement of trapped ions

H. Häffner[1,3], W. Hänsel[1], C. F. Roos[1,3], J. Benhelm[1,3], D. Chek-al-kar[1], M. Chwalla[1], T. Körber[1,3], U. D. Rapol[1,3], M. Riebe[1], P. O. Schmidt[1], C. Becher[1]†, O. Gühne[3], W. Dür[2,3] & R. Blatt[1,3]

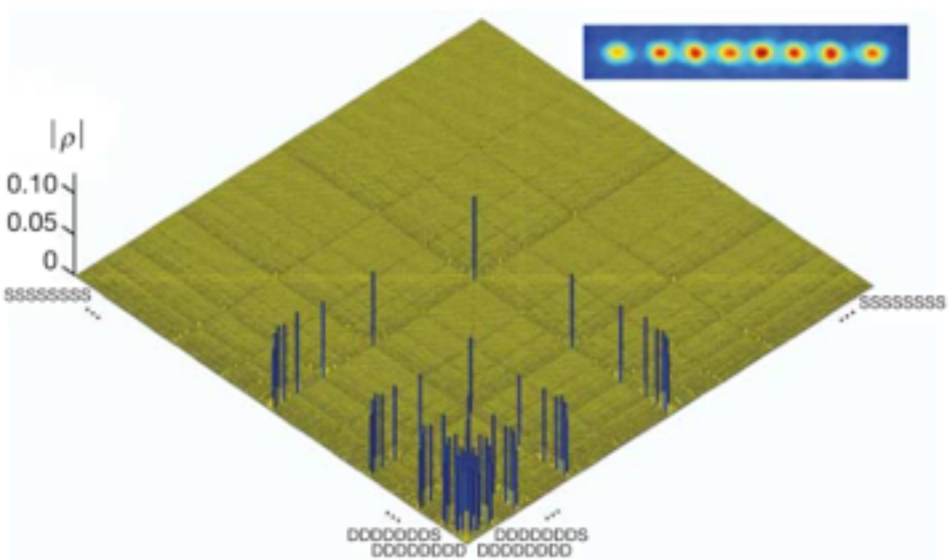
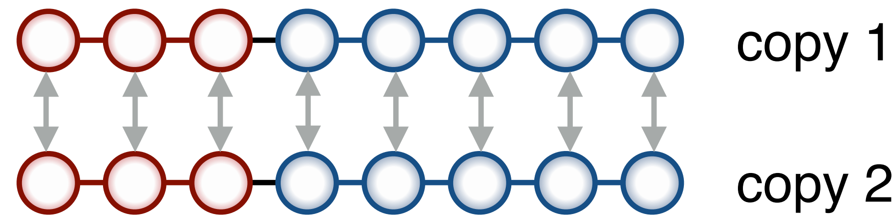


Figure 1 | Absolute values, |ρ|, of the reconstructed density matrix of a |W₈⟩ state as obtained from quantum state tomography. DDDDDDDD…SSSSSSSS label the entries of the density matrix ρ. Ideally,

The probabilistic nature of the measurement process requires an infinite number of measurements for a perfect reconstruction of the density matrix. In order to assess the error introduced by the finite number of measurements (quantum projection noise), we have used a Monte Carlo simulation to create up to 100 comparable data sets.

- expensive: many copies
- tomography for itinerant particles (?)
- [Gaussian states]
- [photons]

$\rho \otimes \rho$



Protocols to measure Rényi entropies

- **a quantum information perspective**

measuring nonlinear functionals of ρ

quantum circuits / computers

A. K. Ekert et al. PRL 2002

- **… and a much more practical protocol**

bosons (& fermions) in 1D/2D
optical lattices

beamsplitter & microscope

hard core bosons = spins in ion traps

Bell state measurements

A. Daley et al, PRL 2012
C. Moura Alves, D. Jaksch, PRL 2004
F. Mintert et al., PRL 2005

Quantum Information Perspective

- Nonlinear functionals of a quantum state can be measured by measuring the expectation value of the unitary operator $V^{(n)}$ on the n-fold copy of the state

$$\mathrm{Tr}\{\rho^n\} = \mathrm{Tr}\{V^{(n)}\rho^{\otimes N}\} \equiv \langle V^{(n)} \rangle$$

Renyi entropy

Measurement of expectation value

$$V^{(n)}|\psi_1\rangle \ldots |\psi_n\rangle = |\psi_n\rangle |\psi_1\rangle \ldots |\psi_{n-1}\rangle$$

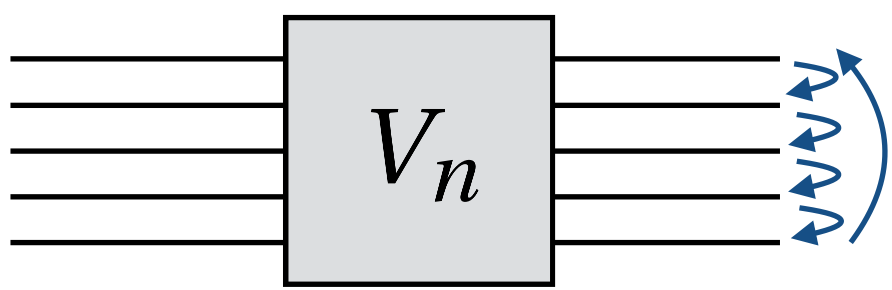
✓ unitary

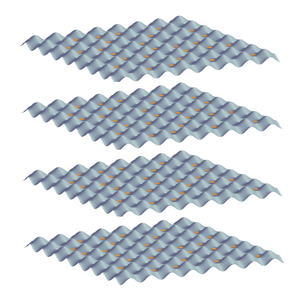
✓ for n=2 swap is also hermitian

shift (or swap) operator

$\rho \otimes \rho \otimes \rho \otimes \rho \otimes \rho$

n copies





A. K. Ekert, C. Moura Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, PRL (2002).

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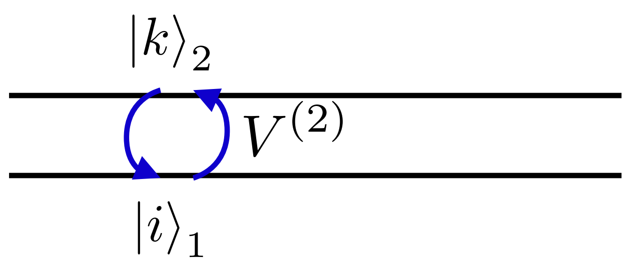
✓ unitary
✓ for n=2 swap is also hermitian

Example n=2:

$\rho_1 \otimes \rho_1$



swap operator

$$V^{(2)}\,|i\rangle_1 \otimes |k\rangle_2 = |k\rangle_1 \otimes |i\rangle_2$$

$$\mathrm{tr}\{V^{(2)}\rho_1 \otimes \rho_2\} = \mathrm{tr}\left\{ V^{(2)} \sum_{ijkl} \rho^{(1)}_{ij}\,\rho^{(2)}_{kl}\,|i\rangle\langle j| \otimes |k\rangle\langle \ell| \right\}$$

expectation value

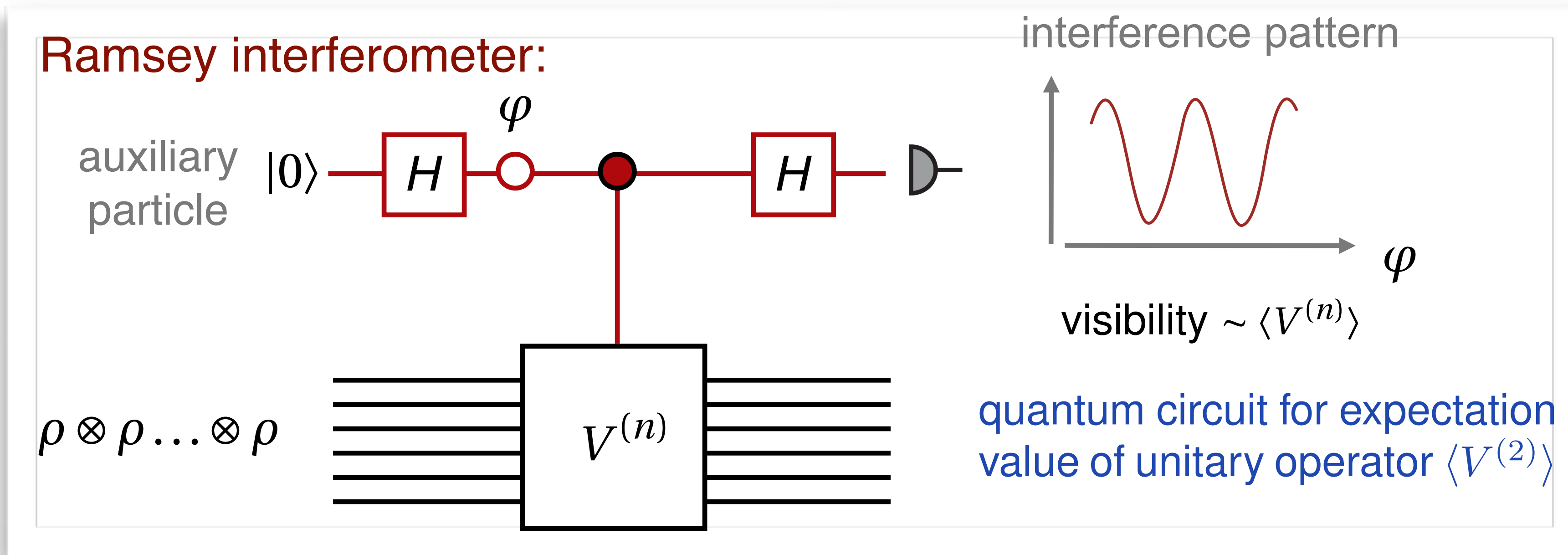
$$= \mathrm{tr}\{\rho_1\rho_2\}$$

Quantum Circuit

- Measurement via quantum network via ancilla qubit and controlled gate between ancilla and the copies of the system.

$$\mathrm{Tr}\{\rho^n\} = \mathrm{Tr}\{V^{(n)}\rho^{\otimes N}\} \equiv \langle V^{(n)} \rangle$$

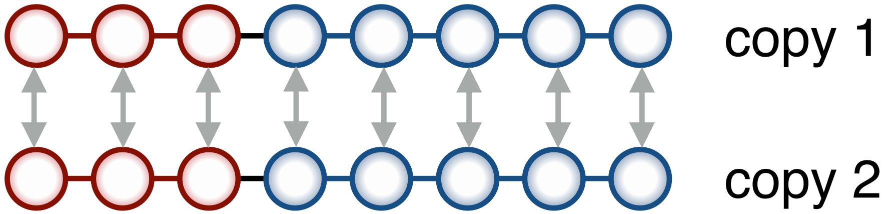
$$V^{(n)}\,|\psi_1\rangle \ldots |\psi_n\rangle = |\psi_n\rangle\,|\psi_1\rangle \ldots |\psi_{n-1}\rangle$$



A. K. Ekert, C. Moura Alves, D. K. L. Oi, M. Horodecki, P. Horodecki, and L. C. Kwek, PRL (2002).

… we need a quantum computer (?)

$\rho \otimes \rho$



Protocols to measure Rényi entropies

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  A. Daley et al, PRL 2012

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  F. Mintert et al., PRL 2005

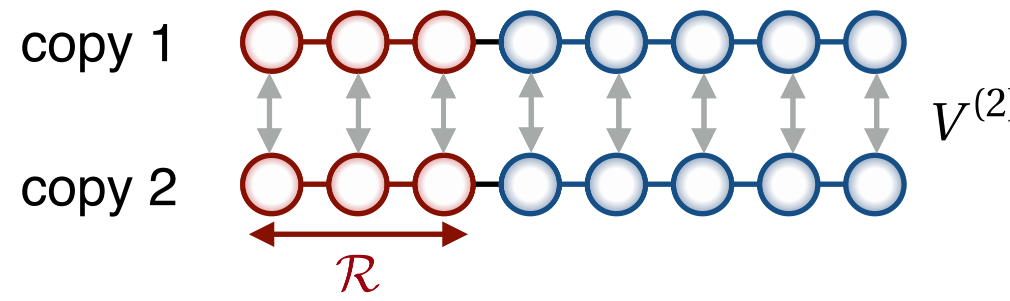
Measurement of Renyi Entropies (n=2)

- **SWAP operator**

$$\mathrm{Tr}\{\rho^2\} = \mathrm{Tr}\{V^{(2)} \rho \otimes \rho\} \equiv \langle V^{(2)} \rangle$$

$$V^{(2)} |\mathbf{n}_1\rangle |\mathbf{n}_2\rangle = |\mathbf{n}_2\rangle |\mathbf{n}_1\rangle$$

boson occupation numbers



Remarks:

- product of local operations $V^{(2)} = \prod_i V^{(2,i)}$
- $V^{(2)}$ hermitian & unitary: eigenvalues $\lambda = +1, -1$

$$V^{(2)} = (+1)\, P_+ + (-1)\, P_-$$

symmetric antisymmetric subspace (copy 1 $\leftrightarrow$ 2)

Measure expectation values of projection operators onto (anti)symmetric subspace (with respect to exchange of copies)

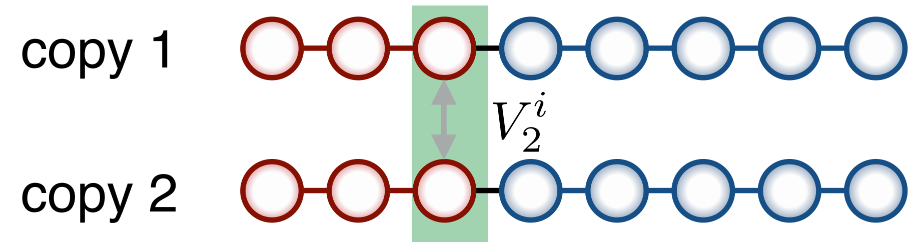
- **identify symmetric and antisymmetric subspaces of the SWAP operator**

V_2^i : swap operator

$a_{i,1} \leftrightarrow a_{i,2}$

$+1$

-1



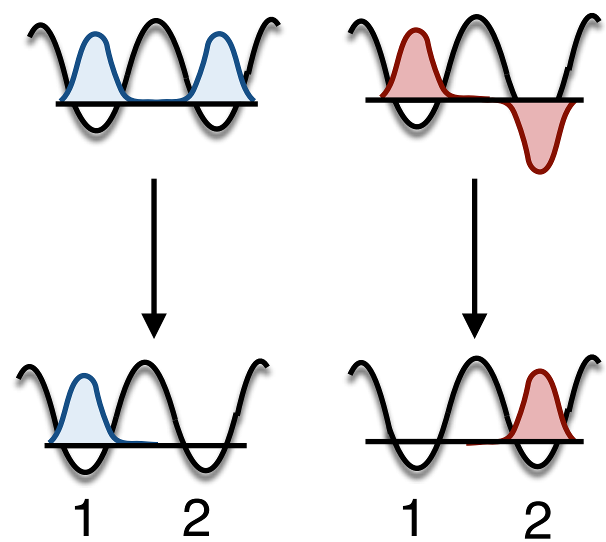
eigenstates

eigenvalue

$$(a^\dagger_{i,1} + a^\dagger_{i,2})^n (a^\dagger_{i,1} - a^\dagger_{i,2})^m |\mathrm{vac}\rangle \qquad (-1)^m$$

- **50/50 beamsplitter**

$$(a^\dagger_{i,1})^n (a^\dagger_{i,2})^m |\mathrm{vac}\rangle$$



(quantum) measurement of V_2^i is simply a measurement of occupation numbers (modulo 2) after a 50/50 beam splitter.

This leads to a protocol, where beam splitter operations and a microscope are sufficient.

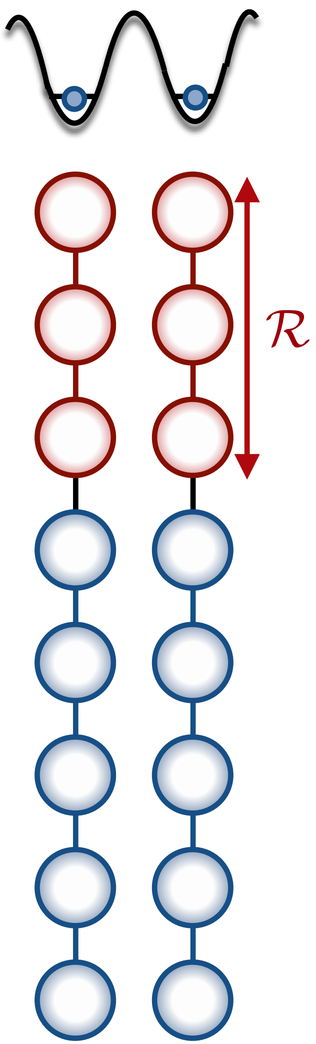
Note: protocol can be generalized to n

"The Recipe": for n=2 (Bosons)

- Renyi entropy $\mathrm{Tr}\{\rho_{\mathcal{R}}^2\} = \mathrm{Tr}\{V_2^{\mathcal{R}} \rho_{\mathcal{R}} \otimes \rho_{\mathcal{R}}\} \equiv \langle V_2^{\mathcal{R}} \rangle$

2 copies

Eigenvalues: ± 1



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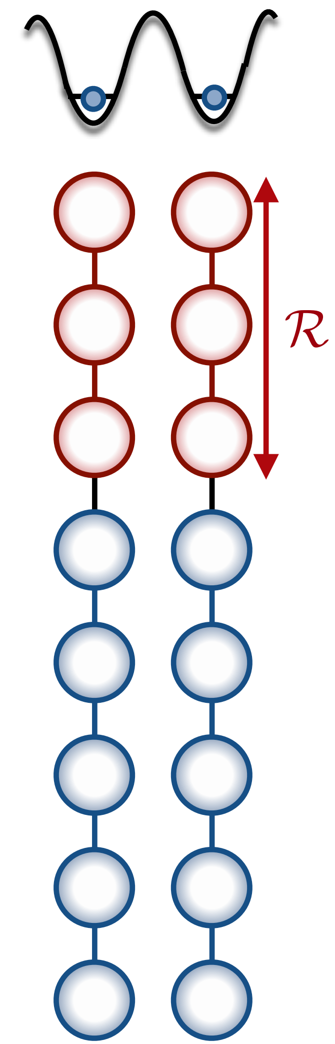
Bosons in 1D optical lattices:

- freeze the motion in the axial direction



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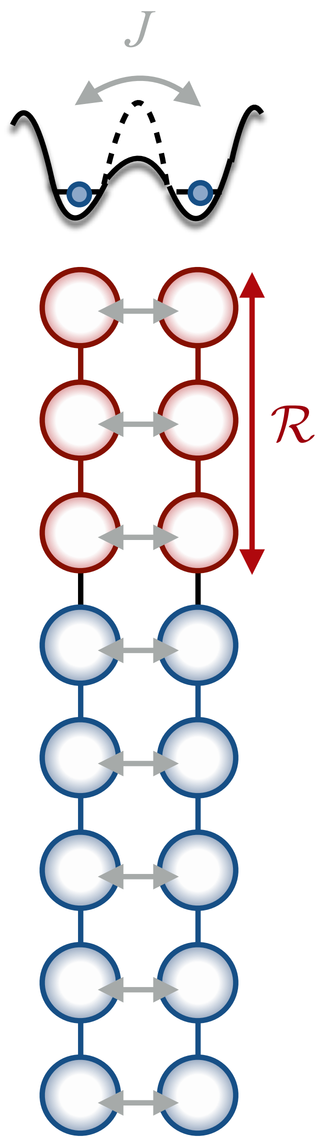
Bosons in 1D optical lattices:

- freeze the motion in the axial direction
- **tunneling** between the two copies using a **superlattice**

  (*turn interaction off!*)

$$a_{j,1} \to \frac{1}{\sqrt{2}}(a_{j,1} + a_{j,2}), \quad a_{j,2} \to \frac{1}{\sqrt{2}}(a_{j,2} - a_{j,1})$$

single particle operations



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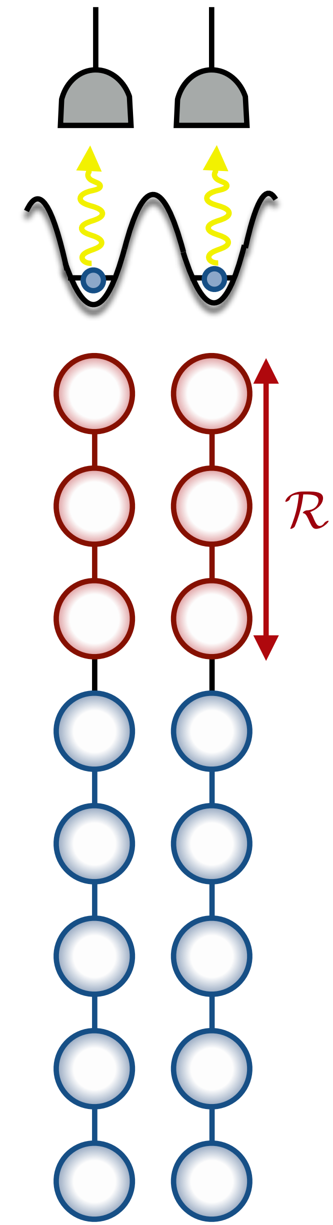
- measure **site resolved** atom number

$\sum_{i\in\mathcal{R}} n_{i,2}$	$V_2^{\mathcal{R}}$
even	$+1$
odd	-1

quantum gas microscope



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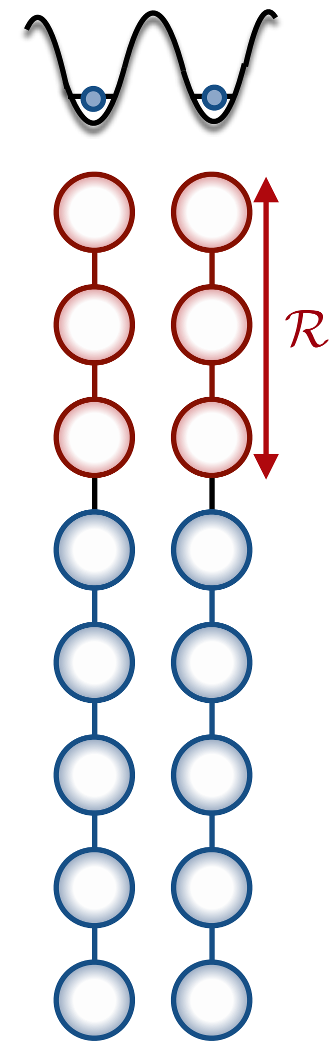

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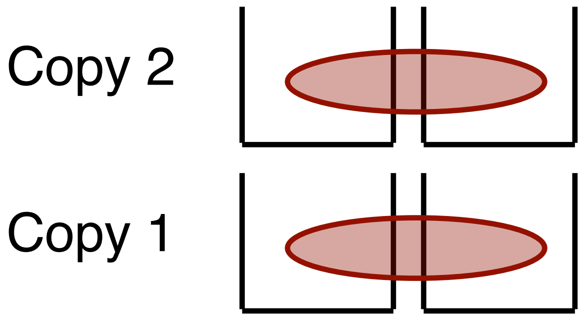
- measure **site resolved** atom number
- repeat

$$\mathrm{Tr}\{\rho_{\mathcal{R}}^2\} = \langle V_2^{\mathcal{R}} \rangle = \langle (-1)^{\sum_{i \in \mathcal{R}} n_{i,2}} \rangle_{\mathrm{measure}}$$

Example: Detecting a Superfluid (two sites)



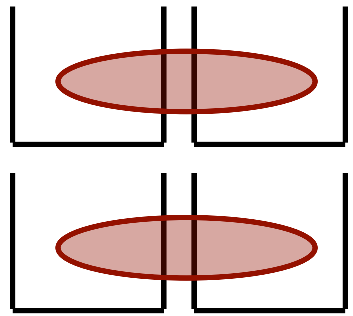
$$(a^\dagger_{1,1} + a^\dagger_{2,1})^2 (a^\dagger_{1,2} + a^\dagger_{2,2})^2 |\mathrm{vac}\rangle$$

SF 1 SF 2

Example: Detecting a Superfluid (two sites)

Copy 2

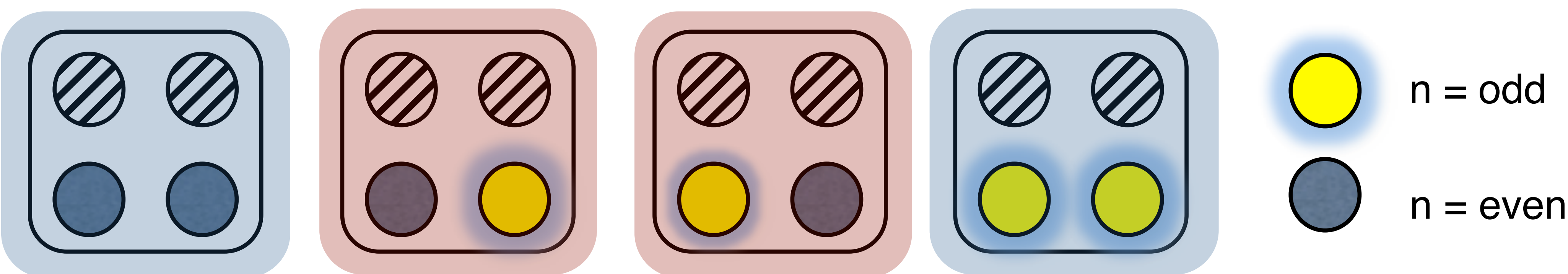
Copy 1



$$(a^\dagger_{1,1} + a^\dagger_{2,1})^2 (a^\dagger_{1,2} + a^\dagger_{2,2})^2 |\mathrm{vac}\rangle$$

SF 1 SF 2

Possible read outs (after beam-splitter):



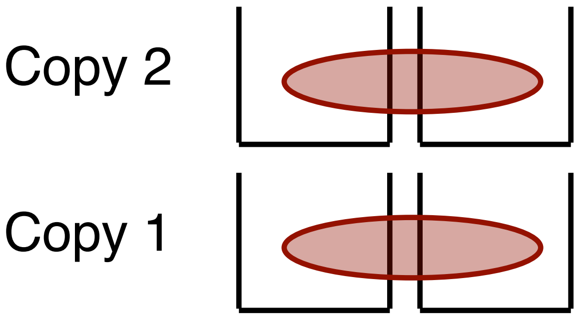
Probabilities:

$p = \frac{11}{16}$ $p = 0$ $p = 0$ $p = \frac{5}{16}$

$$\mathrm{Tr}\{\rho^2\} = \langle V_2^{\{1,2\}} \rangle = +1 \times \left(\frac{11}{16} + \frac{5}{16} \right) - 1 \times (0 + 0) = 1$$

Pure

Example: Detecting a Superfluid (two sites)



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SF 1 SF 2

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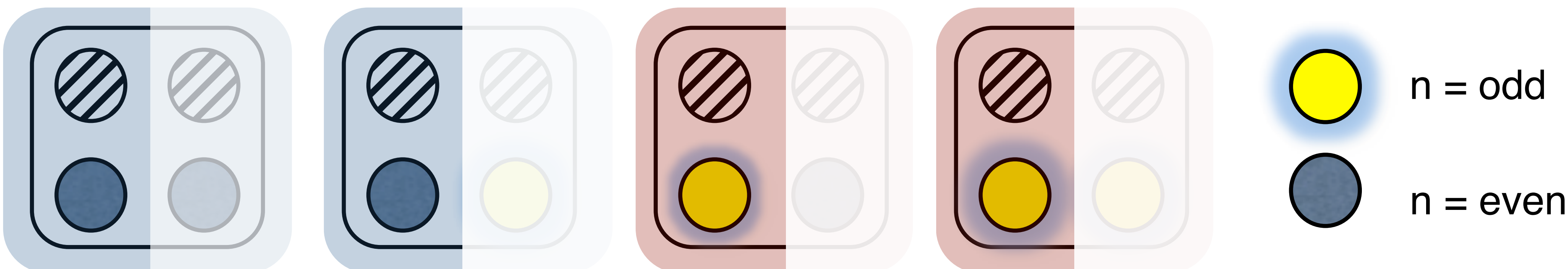


Probabilities:

$p = \frac{11}{16}$ $p = 0$ $p = 0$ $p = \frac{5}{16}$

$$\mathrm{Tr}\{\rho_1^2\} = \langle V_2^{\{1\}} \rangle = +1 \times \left(\frac{11}{16} + 0 \right) - 1 \times \left(0 + \frac{5}{16} \right) = \frac{3}{8}$$

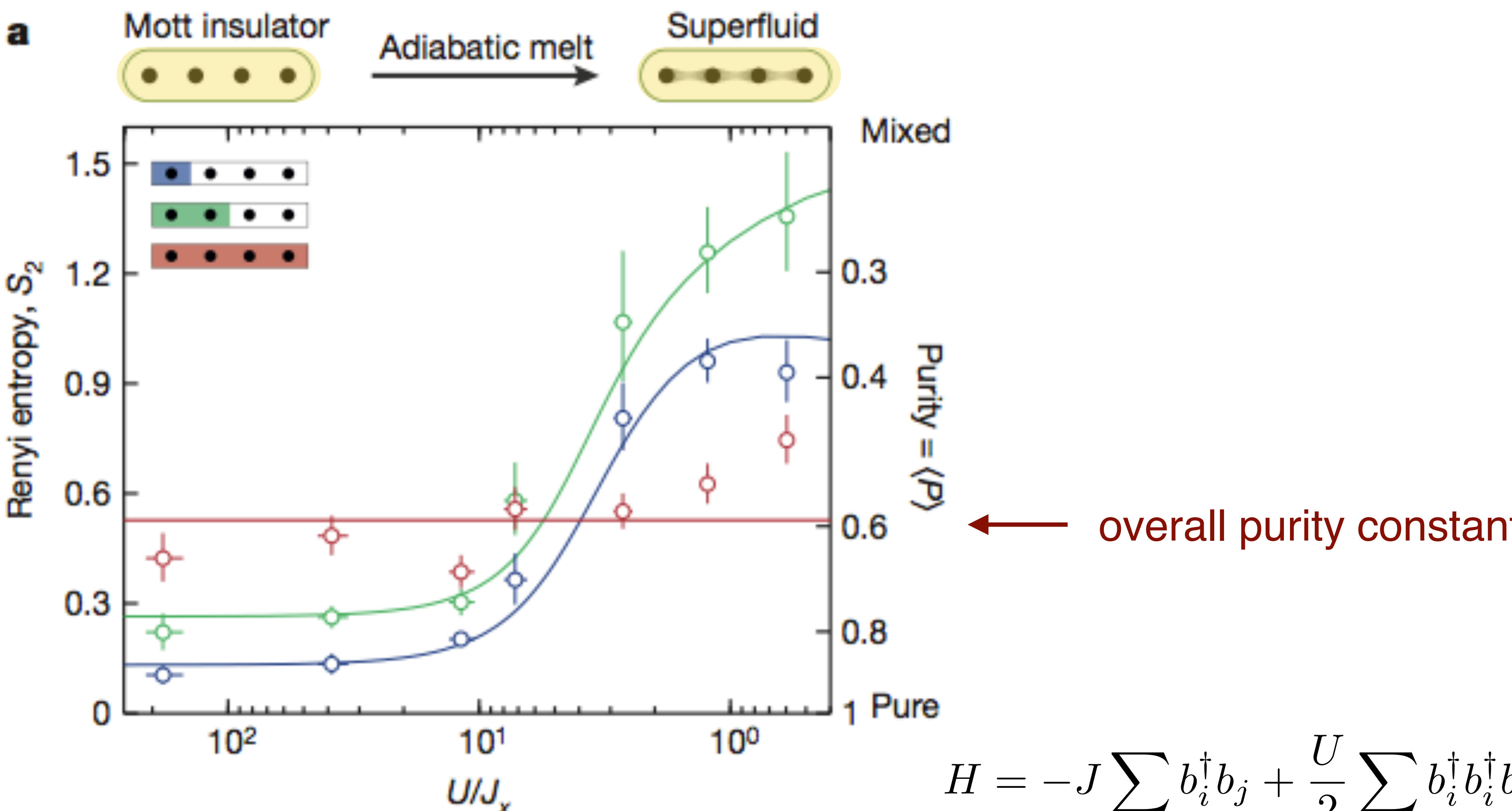
Mixed

Measuring entanglement entropy in a quantum many-body system

doi:10.1038/nature15750

Rajibul Islam[1], Ruichao Ma[1], Philipp M. Preiss[1], M. Eric Tai[1], Alexander Lukin[1], Matthew Rispoli[1] & Markus Greiner[1]

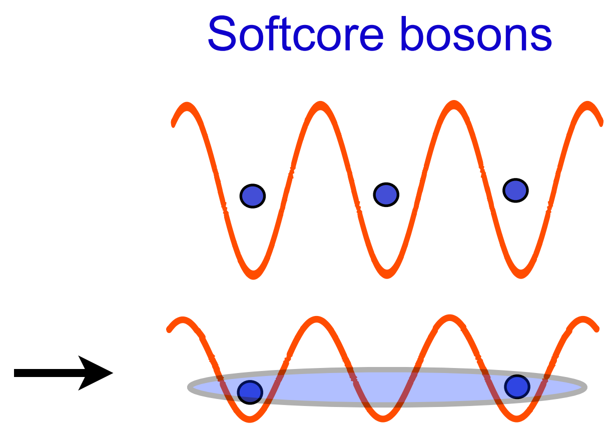
- **Entanglement in the ground state of the Bose-Hubbard model**

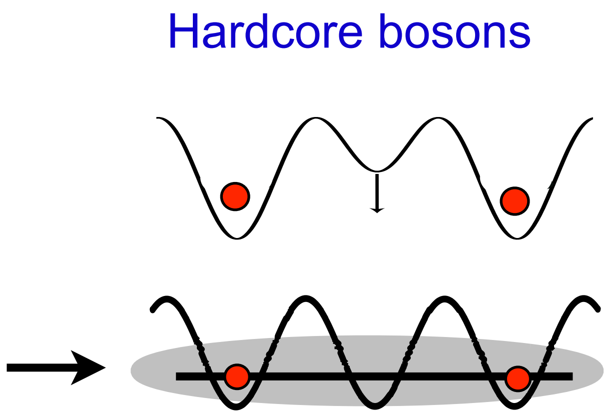


$$H = -J \sum_{i,j} b_i^\dagger b_j + \frac{U}{2} \sum_i b_i^\dagger b_i^\dagger b_i b_i$$

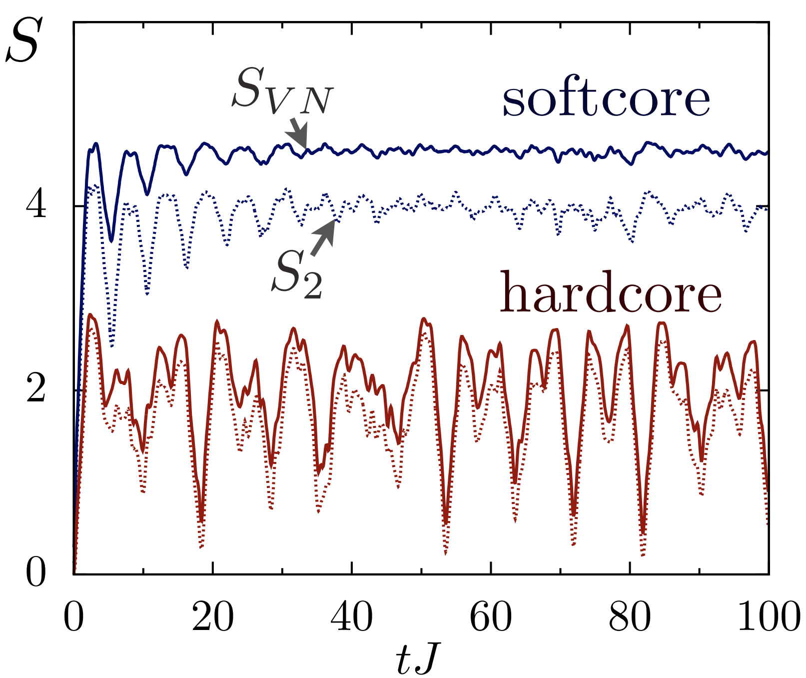
Bose-Hubbard Hamiltonian

Theory: Quantum Quenches





- Bose-Hubbard U/J=10 to U/J=1 quench
- Odd sites initially filled



Softcore bosons, small system (N=M=8):

- Increasing entanglement, saturates (thermalization)

Hardcore bosons (M=8)

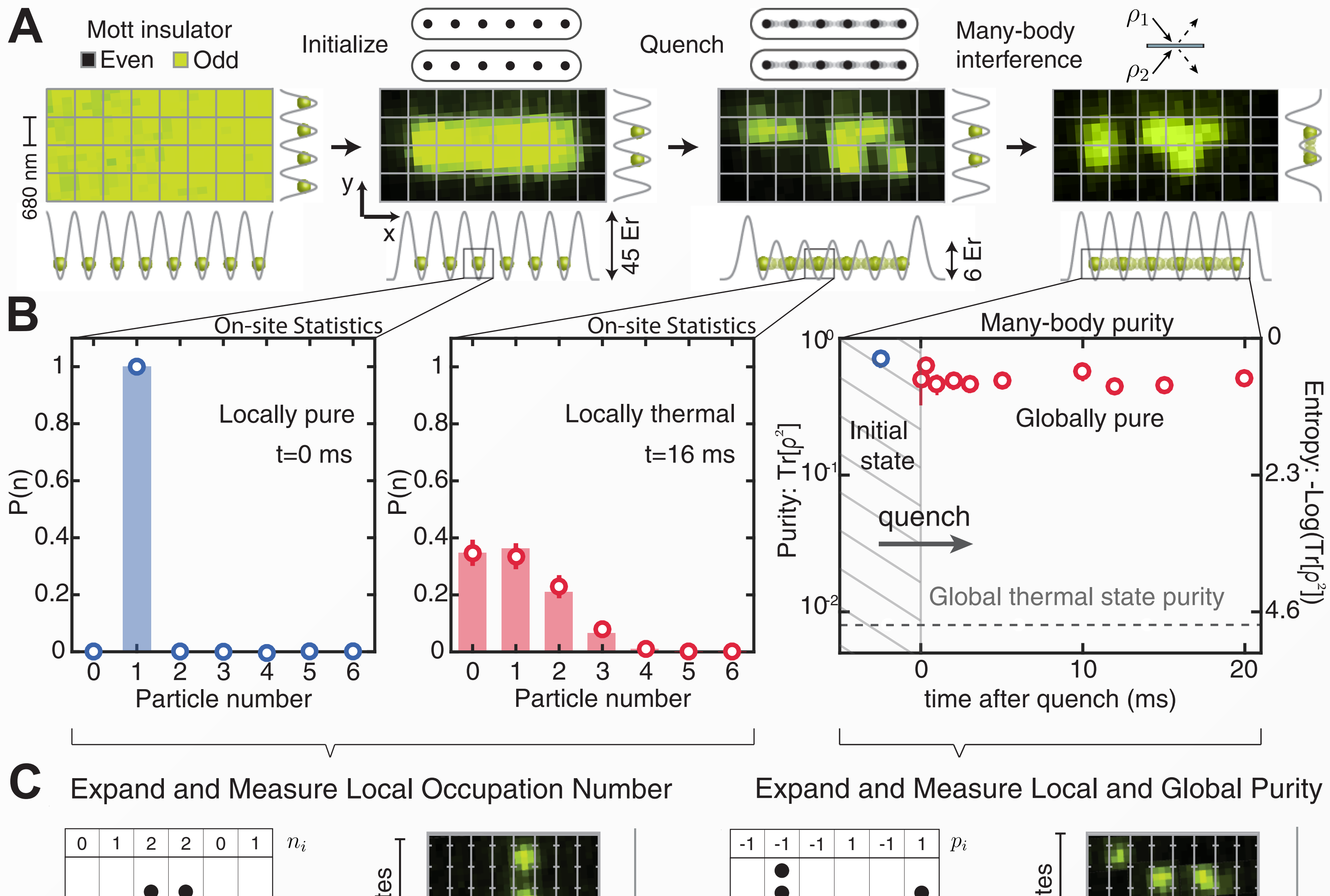
- Initial growth of entanglement, then oscillations (integrable system)

tDMRG calculations by
A Daley & J Schachenmayer

Quantum thermalization through entanglement in an isolated many-body system

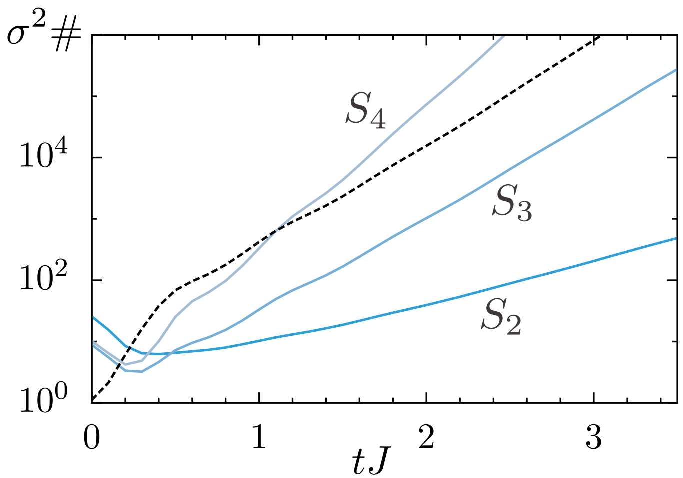
A. M. Kaufman, M. E. Tai, A. Lukin, M. Rispoli, R. Schittko, P. M. Preiss, and M. Greiner*

arXiv: 1603.04409



Remarks

- **number of measurements for a given precision**

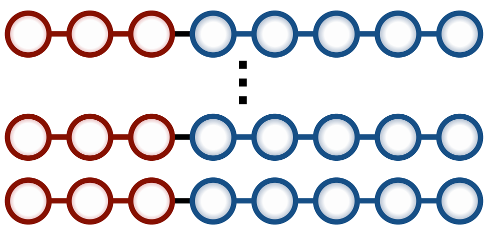


- Larger n requires more measurements for same precision
- However, combination of n=2 and higher n gives stronger bound on von Neumann entropy, e.g., (dashed line in measurement)

$$S_{VN}(\rho) \geq 2S_2(\rho) - S_3(\rho)$$

- **role of imperfections …**

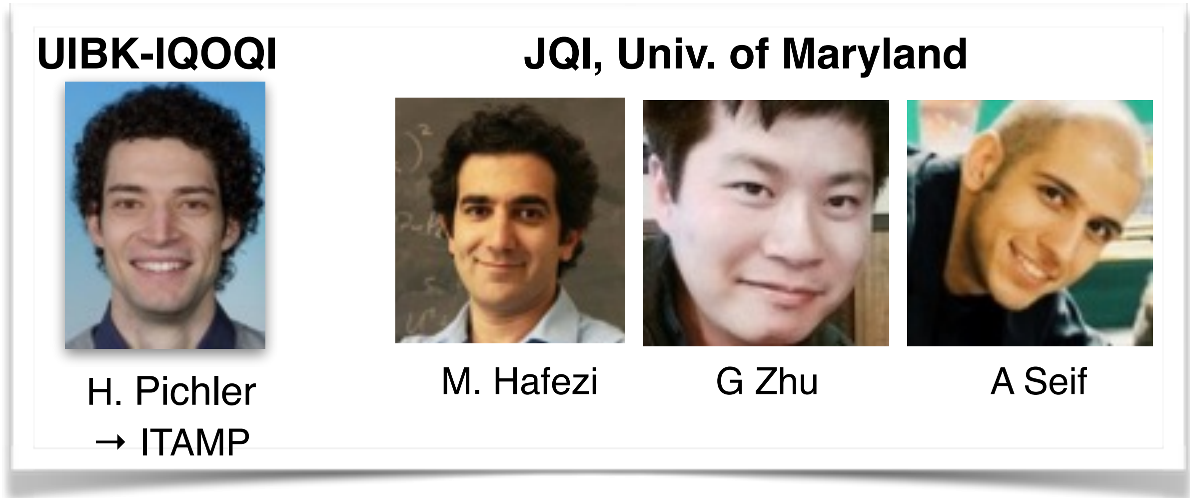
Extensions



- **Higher order** Renyi entropies:

$$U_n^{FT} : a_{j,k} \rightarrow \frac{1}{\sqrt{n}} \sum_{\ell=1}^{n} a_{j,\ell} e^{i\frac{2\pi n}{n}(k-1)(\ell-1)}$$

- **Fermions**: same experimental procedure, different interpretation of measurement record



Measuring the *Entanglement Spectrum*

- Ramsey interferometry: $\rho_\alpha = \mathrm{Tr}_{\mathcal{H}_\beta}\{\rho\} = \sum_k \lambda_k |\phi_k\rangle\langle\phi_k|$

Entanglement spectrum

H. Pichler, G. Zhu, A. Seif, PZ, and M Hafezi, arXiv May 2016

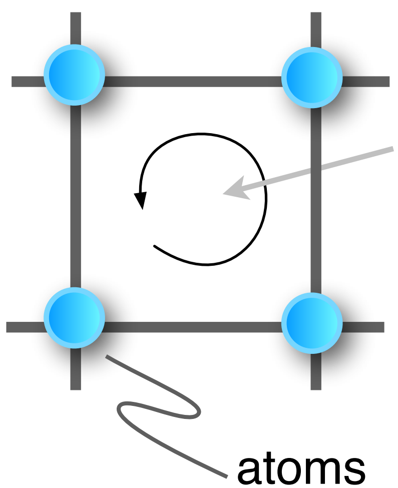
$$\mathrm{Tr}\rho_\alpha^n = \sum_k \lambda_k^n$$

Special Topic 2:

From Synthetic Static to Dynamical Gauge Fields

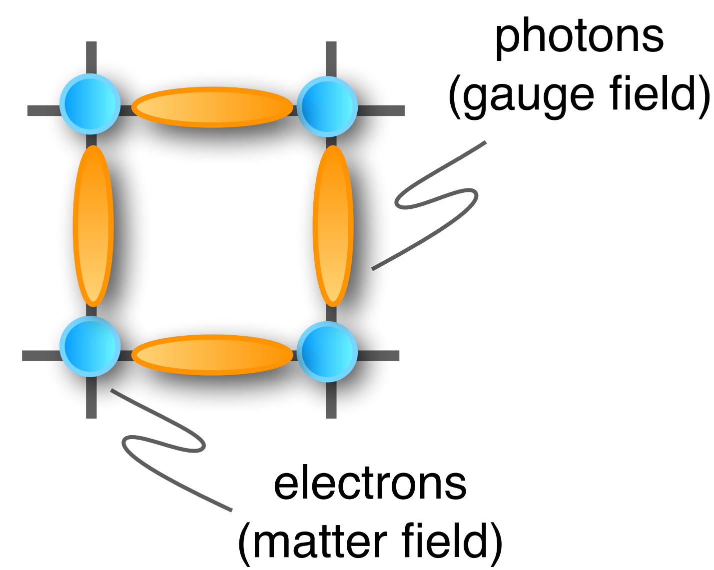
Synthetic Gauge Fields [Static]

Lattice Gauge Theory



$$\Phi = \int d^2\vec{f} \cdot \vec{\nabla} \times \vec{A}$$

flux

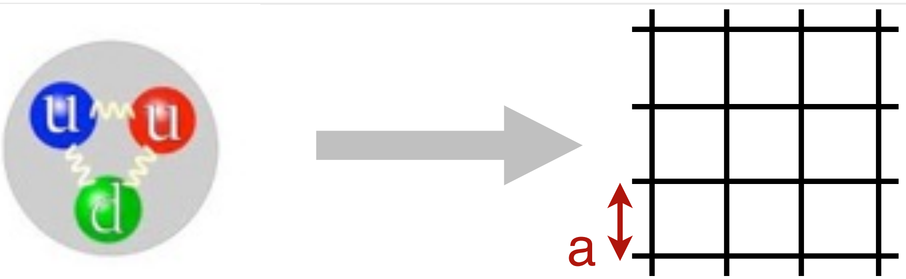


- condensed matter
- high-energy physics

… lattice gauge theories [in particle physics]

K. Wilson 1974

- Gauge theories on a discrete lattice structure
- **Fundamental gauge symmetries:** standard model (every force has a gauge boson)



quarks and gluons

lattice

non-perturbative approach to fundamental theories of matter (e.g. QCD)
→ classical statistical mechanics

Classical Monte Carlo simulations:

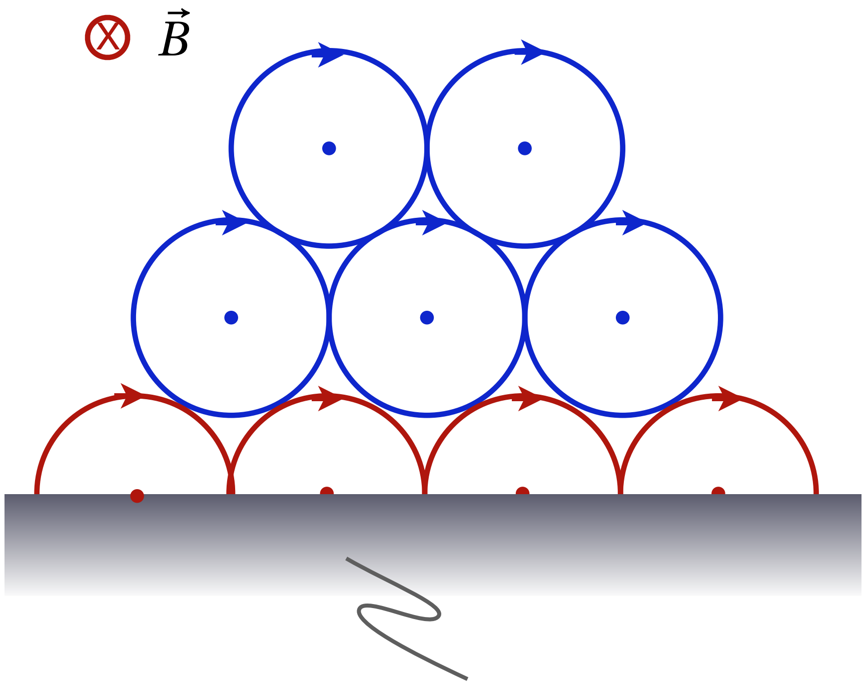
achievements	issues
• first evidence of quark-gluon plasma • ab initio estimate of proton mass • entire hadronic spectrum	Sign problem in its various flavors: • finite density QCD (=fermions) • real time evolution

Quantum simulation (with atoms)? … toy models & simple phenomena

Charged Particle in *Static* Magnetic Field

- **Classical Physics**

cyclotron orbits in B-field

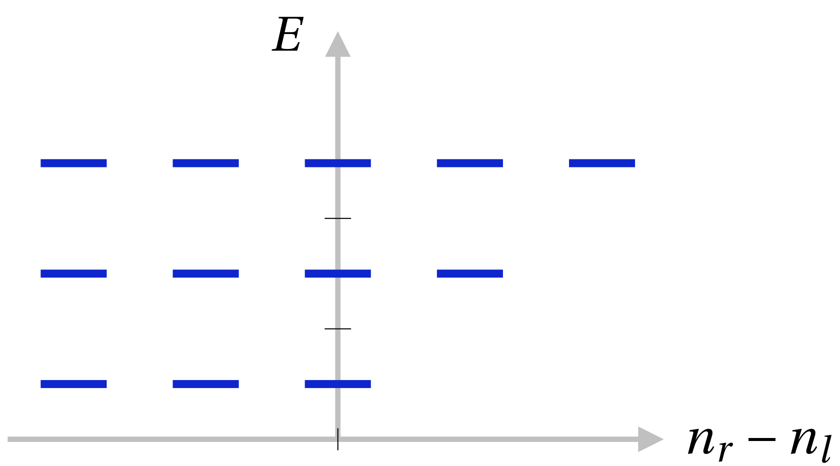


edge states

"skipping orbits"

- **Quantum Physics**

Landau levels



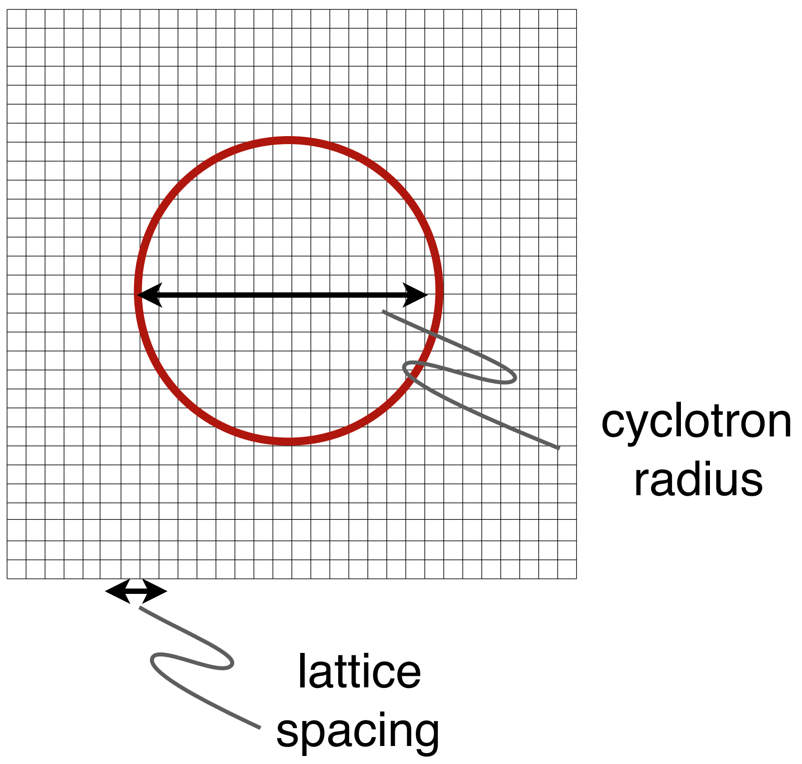
Condensed Matter Physics

- Quantum Hall effect
- Fractional Quantum Hall effect

- topology

Charged Particle in *Static* Magnetic Field

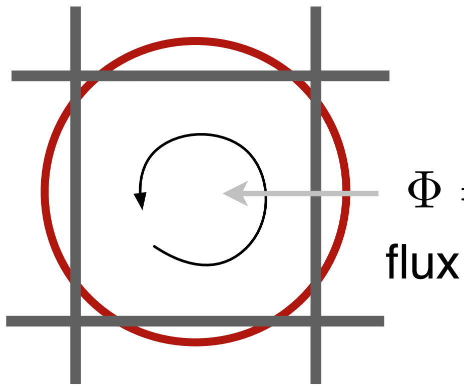
- **… on a Lattice**



Charged Particle in *Static* Magnetic Field

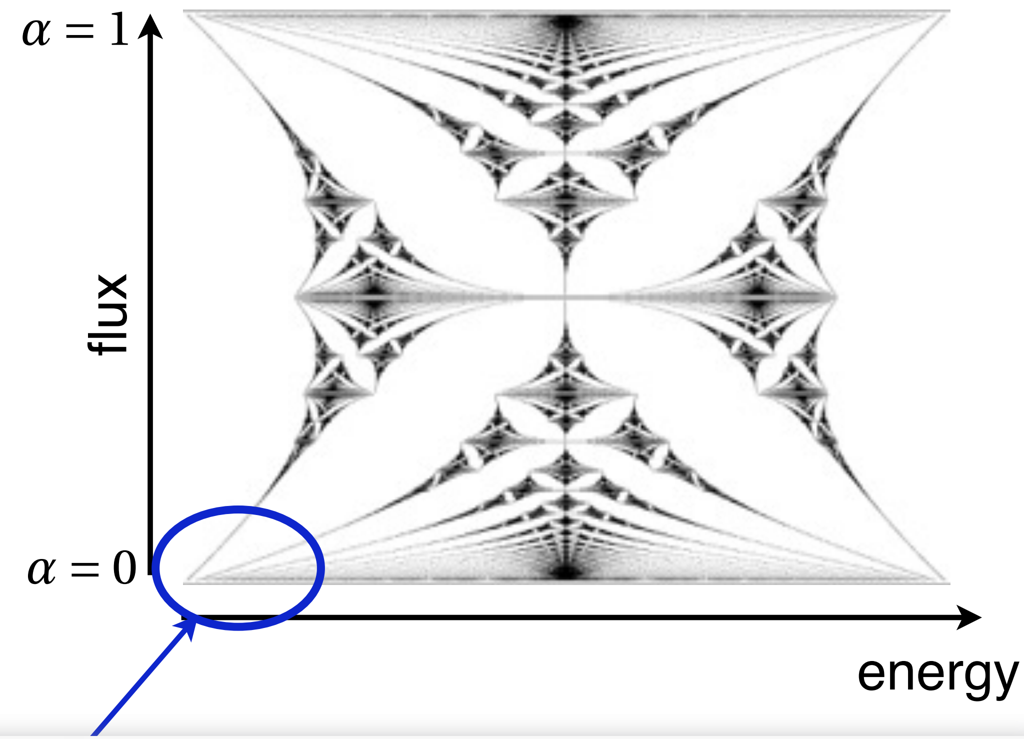
D. Hofstadter

- **… on a Lattice**



$$\Phi = \int d^2\vec{f} \cdot \vec{B}$$

particles hopping around a plaquette acquire a phase $2\pi\alpha$



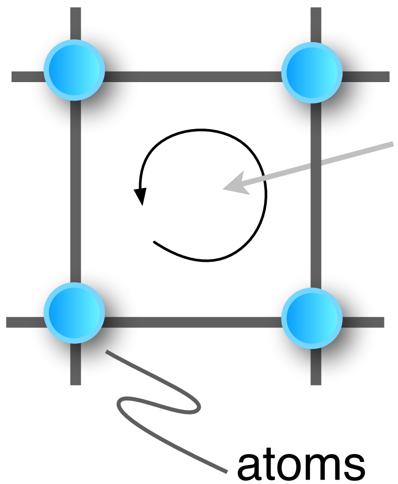
PHYSICAL REVIEW B VOLUME 14, NUMBER 6 15 SEPTEMBER 1976

Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields*

Douglas R. Hofstadter†
Physics Department, University of Oregon, Eugene, Oregon 97403
(Received 9 February 1976)

Static vs. *Dynamical* Gauge Fields on a Lattice: U(1)

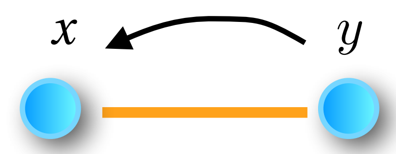
- **c-number / static gauge fields**



$$\Phi = \int d^2\vec{f} \cdot \vec{\nabla} \times \vec{A}$$

flux

“synthetic gauge fields”



$$H = -t\psi_x^\dagger e^{i\varphi_{xy}} \psi_y + \text{h.c.}$$

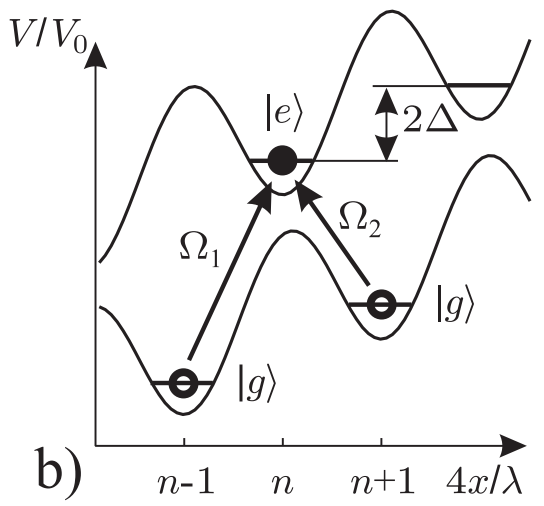
phase $\varphi_{xy} = \int_x^y d\vec{l} \cdot \vec{A}$

U(1) (abelian)

Review: J. Dalibard et al. Rev. Mod. Phys. (2011)

Static Synthetic Gauge Fields with Cold Atoms

- **c-number / static synthetic gauge fields for atoms**



D. Jaksch and P. Zoller, New J. Phys. **5**, (2003).

Selected for a Viewpoint in *Physics*
PRL **111,** 185301 (2013) PHYSICAL REVIEW LETTERS week ending 1 NOVEMBER 2013

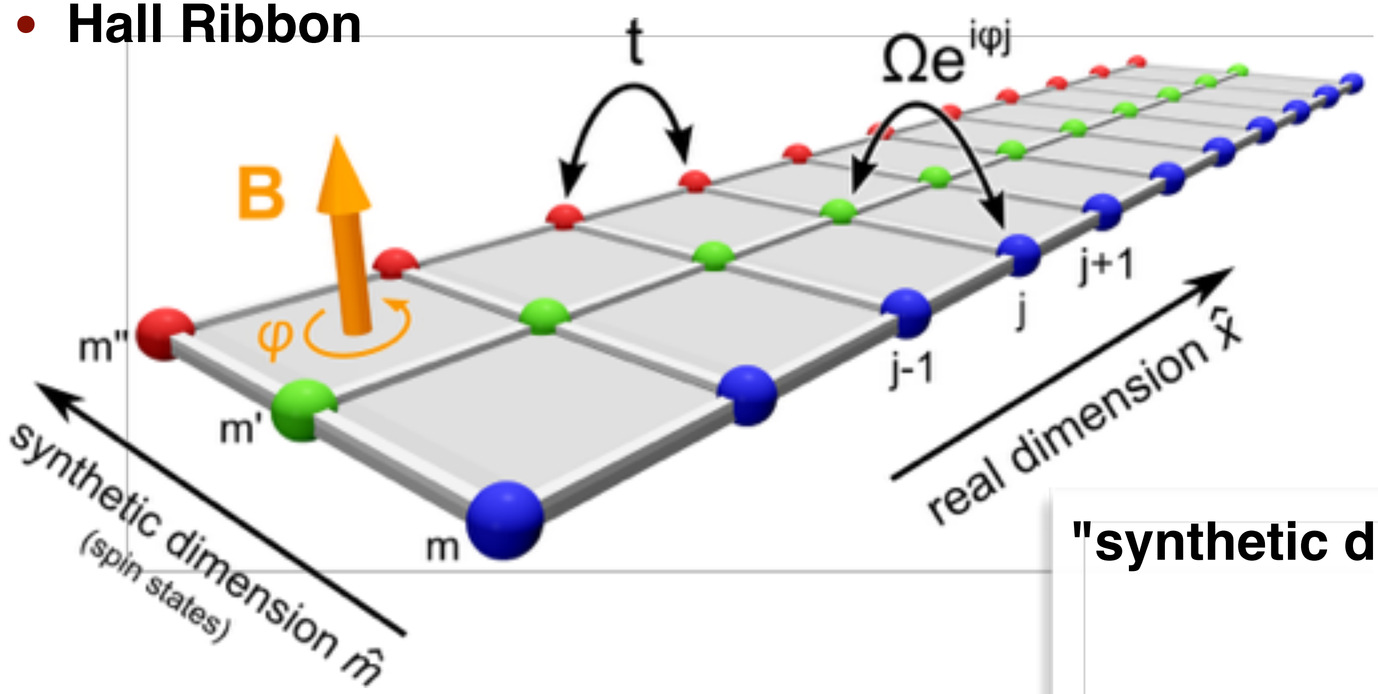
Realization of the Hofstadter Hamiltonian with Ultracold Atoms in Optical Lattices

M. Aidelsburger,[1,2] M. Atala,[1,2] M. Lohse,[1,2] J. T. Barreiro,[1,2] B. Paredes,[3] and I. Bloch[1,2]

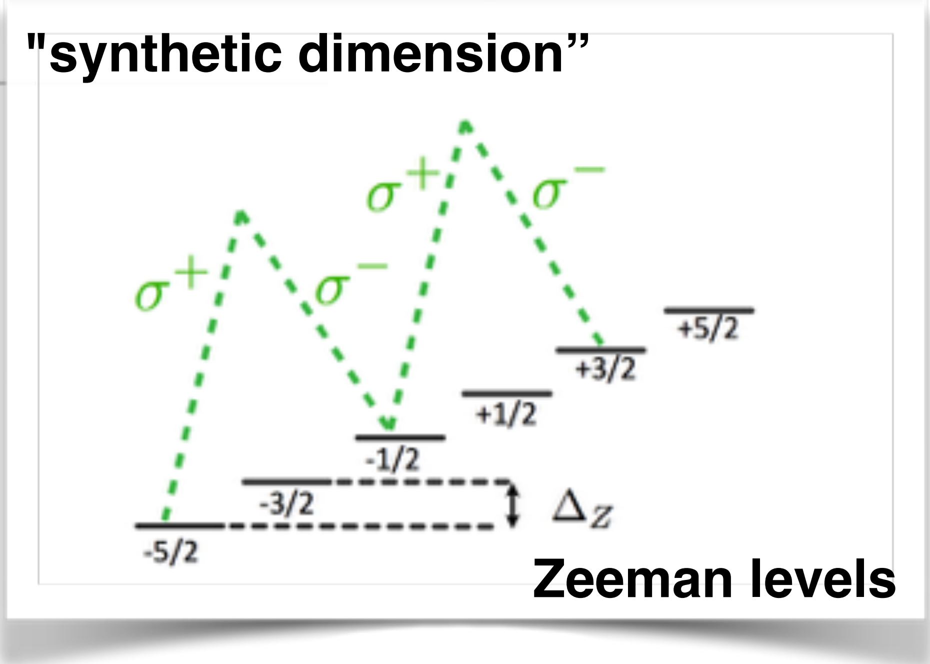
Synthetic [Classical] Gauge Fields: Fermionic Atoms

L. Fallani M. Inguscio
LENS, Florence

- **Hall Ribbon**



$$H = \sum_j \sum_m -t(c^\dagger_{j,m} c_{j+1,m} + \text{h.c.}) + \sum_j \sum_m \frac{\Omega}{2}(e^{i\varphi j} c^\dagger_{j,m} c_{j,m+1} + \text{h.c.})$$

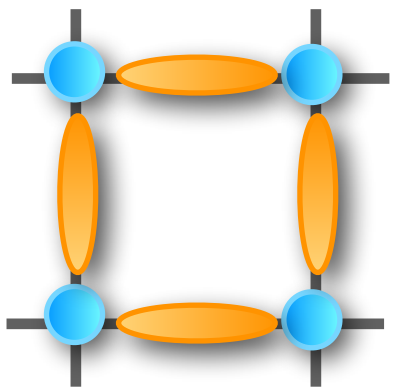


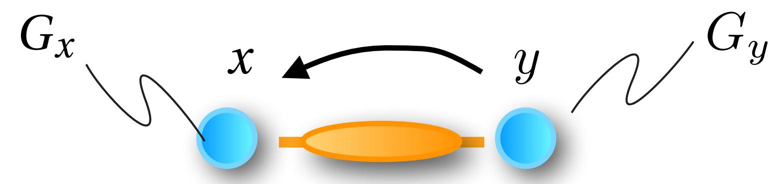
experiment: L. Fallani, M. Ingusico et al., LENS theory: M. Dalmonte, M. Rider, PZ

Static vs. Dynamical Gauge Fields on Lattices: U(1)

- **dynamical gauge fields**

  particles hopping around a plaquette assisted by link degrees of freedom





$$H = -t\psi_x^\dagger U_{xy}\psi_y + \text{h.c.} + \ldots$$

gauge *(local)* symmetry U(1)

gauge bosons $\quad U_{xy} \xrightarrow{V} e^{i\alpha_x} U_{xy} e^{-i\alpha_y},$

fermions $\quad \psi_x \xrightarrow{V} e^{i\alpha_x}\psi_x$

unitary trafo: $\quad V = \prod_x e^{i\alpha_x G_x}$

generator

***local* conserved quantity**

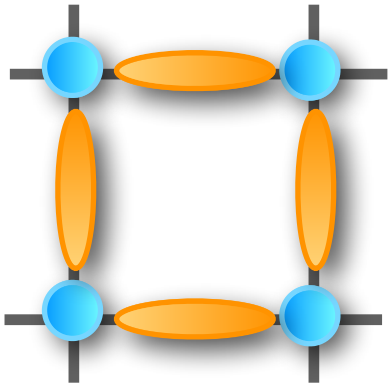
$$[H, G_x] = 0 \ \forall x$$

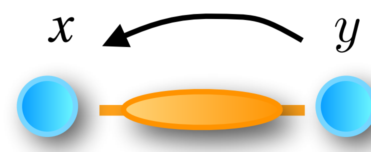
generator of gauge transformation

Static vs. Dynamical Gauge Fields on Lattices: U(1)

- **dynamical gauge fields**

  particles hopping around a plaquette assisted by link degrees of freedom





$$H = -t\psi_x^\dagger U_{xy}\psi_y + \mathrm{h.c.} + \ldots$$

Gauss Law

$$G_x = \psi_x^\dagger \psi_x - \sum_i \left(E_{x,x+\hat{i}} - E_{x-\hat{i},x} \right)$$

matter — electric field operator

$$\rho - \nabla \cdot E = 0$$

local **conserved quantity**

$$[H, G_x] = 0 \;\forall x$$

generator of gauge transformation

Global vs. local (gauge) symmetries?

- **Global symmetries**

Example: particle conservation

$$H = -t \sum_i (c_i^\dagger c_{i+1} + h.c.)$$

Invariant under global transformations

$$c_i \to e^{i\phi} c_i \;\forall i$$

$$[H, \sum_i n_i] = 0$$

Global conserved quantity!

$$N_{\text{TOT}} = \sum_i n_i$$

- **Local (gauge) symmetries**

Example: QED as gauge theory

$$[H, G_x] = 0 \;\forall x$$

Invariant under global transformations

$$\begin{aligned} U_{xy} &\to e^{i\alpha_x} U_{xy} e^{-i\alpha_y} \\ \psi_x &\to e^{i\alpha_x} \psi_x \end{aligned}$$

Local conserved quantity!

$$\rho - \nabla \cdot E = 0$$

Gauge law

$$G_x = \psi_x^\dagger \psi_x - \sum_i \left(E_{x,x+\hat{i}} - E_{x-\hat{i},x} \right)$$

matter

electric field operator

Glossary of lattice gauge theories

A (not too formal) definition of a lattice gauge theory

- **set of fields** acting on the vertices (*matter fields*) and on the links (*gauge fields*)

ψ_x $\quad\quad$ $U_{x,x+1}$

- **set of *generators*,** which define the gauge symmetry, and the physical Hilbert space

$$[G_x, U_{y,\mu}] = (\delta_{x,y+\hat{\mu}} - \delta_{x,y})\, U_{y,\mu}$$

$$G_x|\Psi_{\text{phys}}\rangle = 0$$

Gauss' law

(defines physical space)

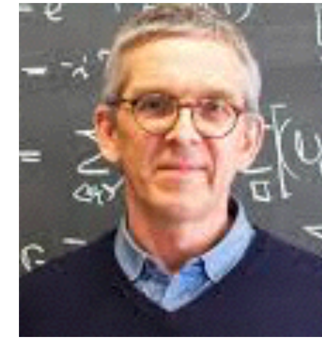
- **Gauge invariant Hamiltonian**:

$$[H, G_x] = 0\ \forall x$$

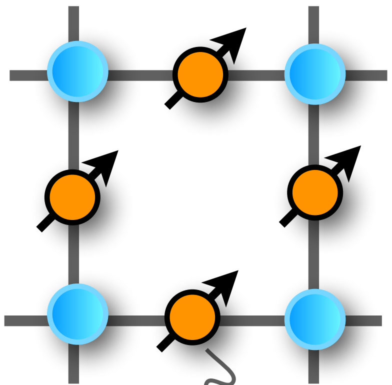
Gauge (local) symmetries

Local conserved quantities

QED with Spins [Quantum Link Model]



Uwe-Jens Wiese



spin S
as quantum link

$$U_{x,x+1} \rightarrow S^{+}_{x,x+1} \qquad E_{x,x+1} \rightarrow S^{z}_{x,x+1}$$

electric flux

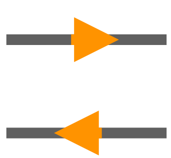
Spin S=½,1,...

quantum link carrying an electric flux

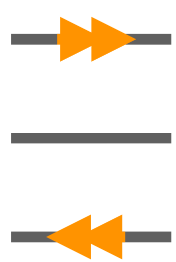
spin-1/2

$E = +1/2$

$E = -1/2$



spin-1

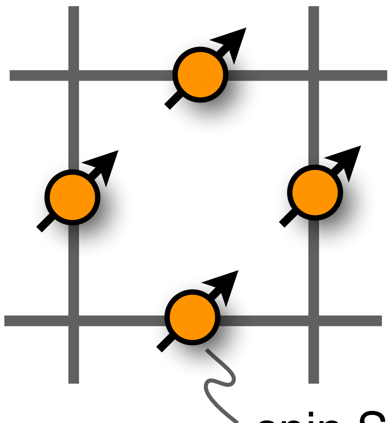


$E = +1$

$E = 0$

$E = -1$

QED with Spins [Quantum Link Model]



spin S
as quantum link

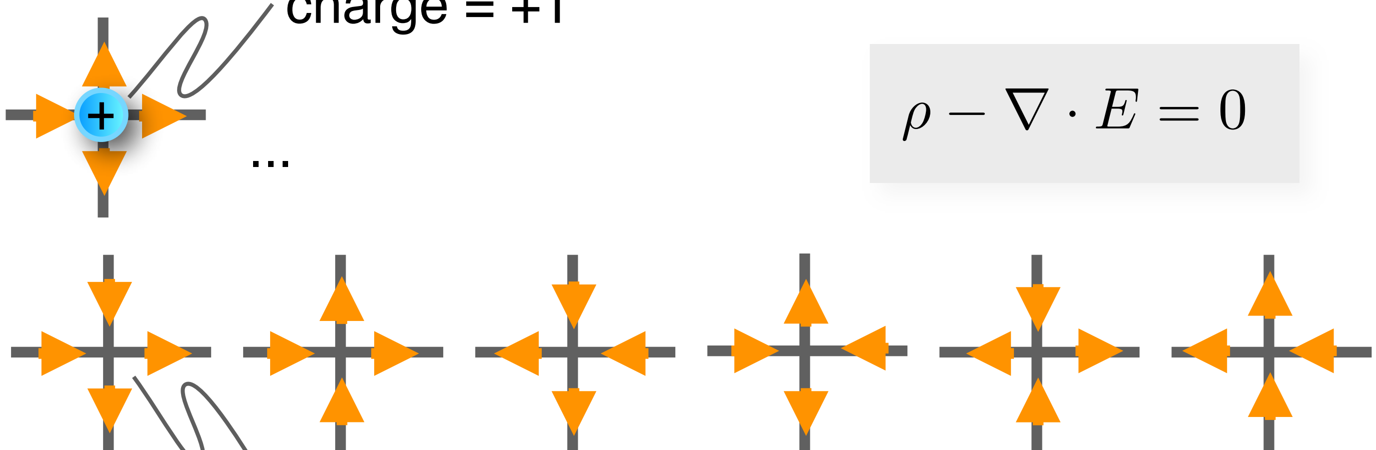
$$U_{x,x+1} \rightarrow S^{+}_{x,x+1} \qquad E_{x,x+1} \rightarrow S^{z}_{x,x+1}$$

electric flux

Spin S=½,1,...

configurations: spin-1/2

charge = +1



charge = 0

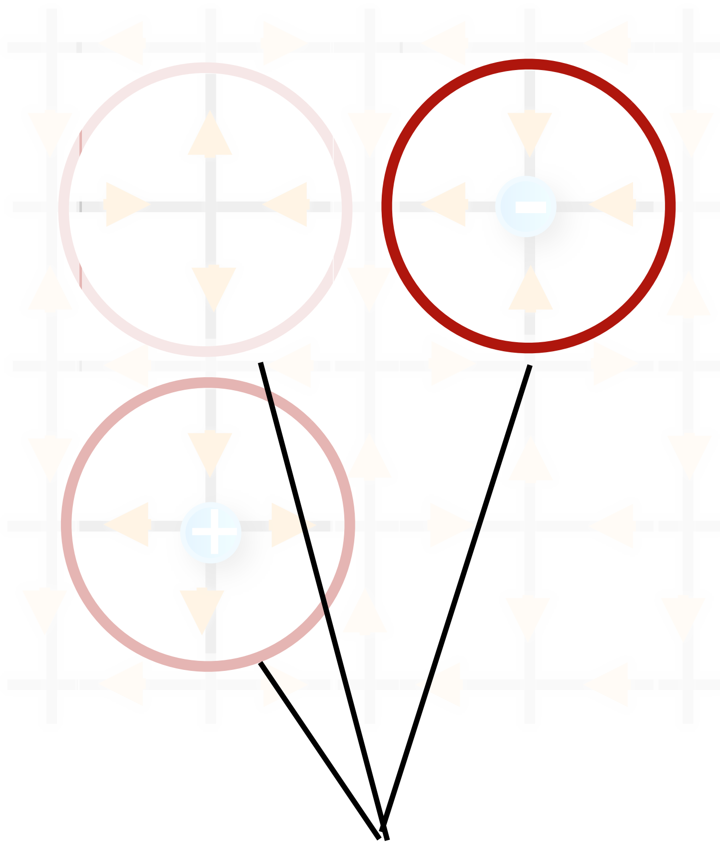
ice rule: *two in, two out*

Gauss Law

$$\rho - \nabla \cdot E = 0$$

ice

superpositions of configurations
satisfying ice rule



$$\rho - \nabla \cdot E = 0$$

Gauss law *as a constraint*

"two in & two out"

Why ICE or SPIN ICE?

What is the Hamiltonian?
see below

… and similar for other / non-Abelian LGT

Cold Atom Implementations
of Dynamical Gauge Fields

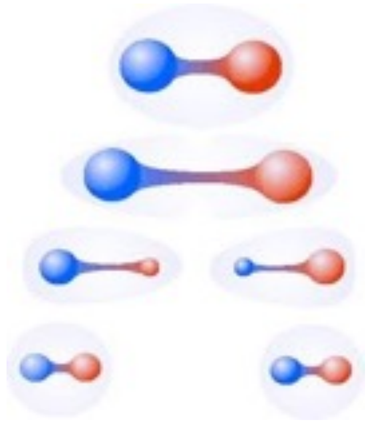
Example 1:

The simplest (meaningful) quantum link model:

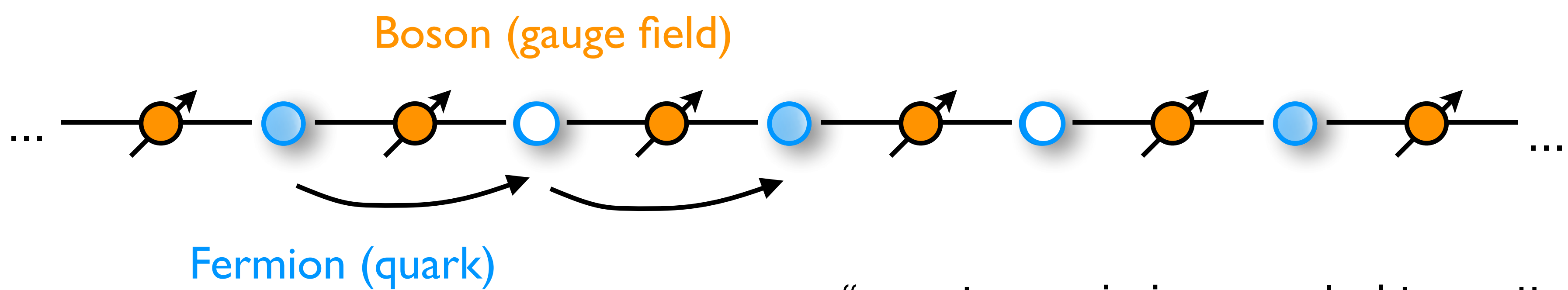
1D Schwinger model

AMO Implementation:

Bose-Fermi Mixtures in Optical Lattices



string breaking



"quantum spin ice coupled to matter"

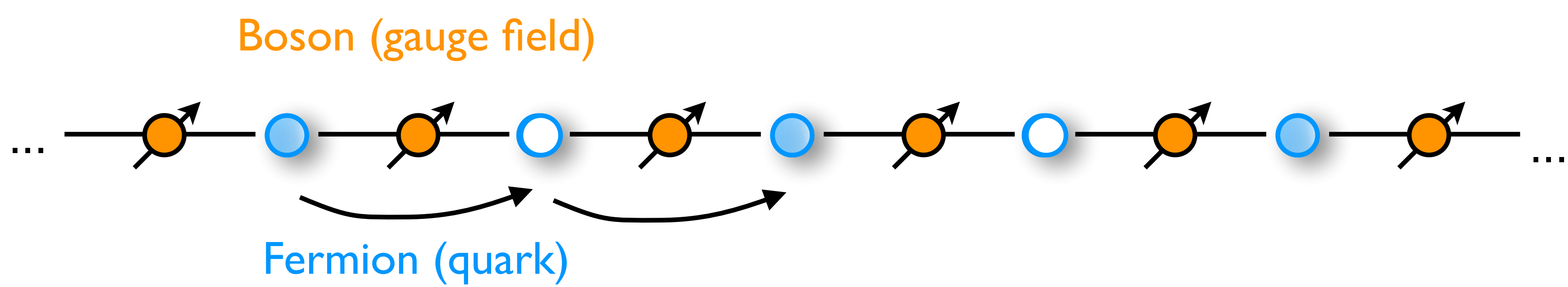
D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U. J. Wiese, and P.Z., PRL 2012

Schwinger Model: U(1) fermions + gauge bosons in 1D

- **Hamiltonian:** staggered fermions in 1D coupled to quantum link spin S

$$H = \frac{g^2}{2}\sum_x E_{x,x+1}^2 - t\sum_x \left[\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

electric flux   hopping   staggered fermions

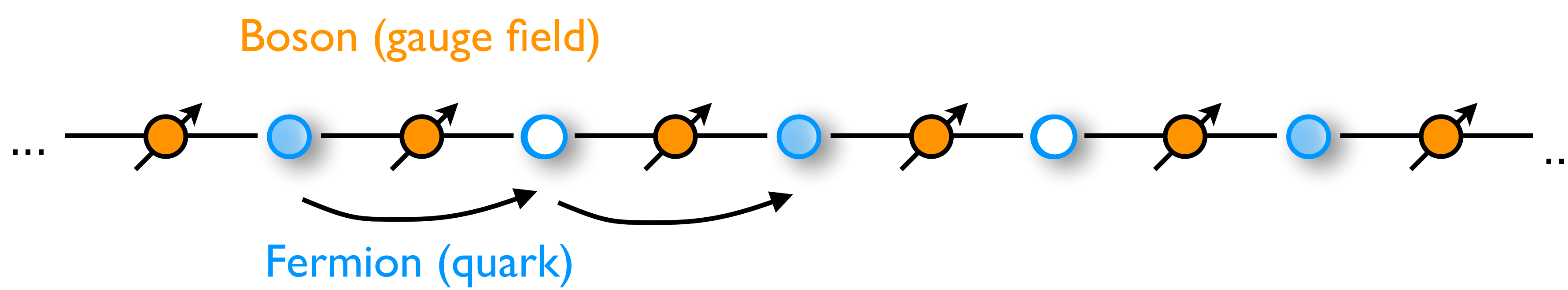


Schwinger Model: U(1) fermions + gauge bosons in 1D

- **Hamiltonian:** staggered fermions in 1D coupled to quantum link spin S

$$H = \frac{g^2}{2}\sum_x ({S^z}_{x,x+1})^2 - t\sum_x \left[\psi_x^\dagger S^+_{x,x+1}\psi_{x+1} + \text{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

electric flux — hopping — staggered fermions



Schwinger Model: U(1) fermions + gauge bosons in 1D

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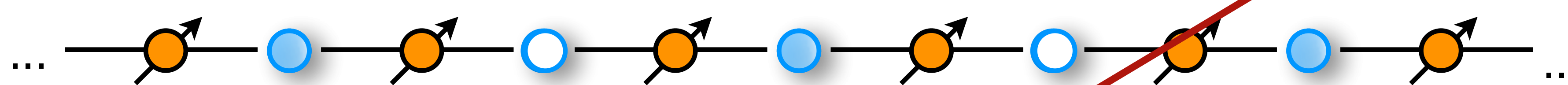
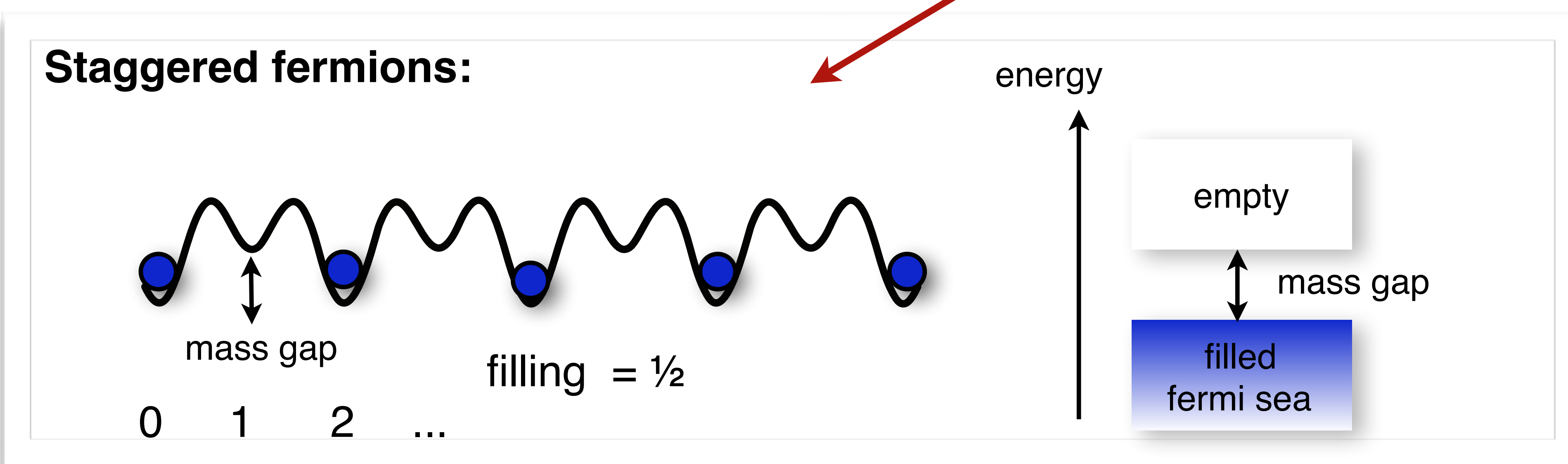
electric flux — hopping — staggered fermions



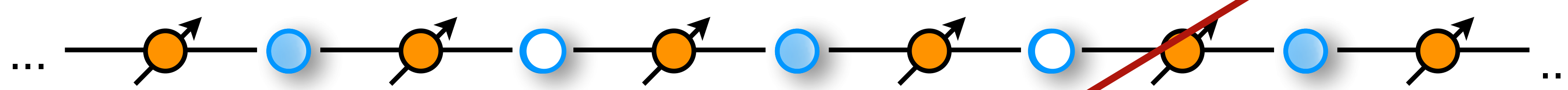


Schwinger Model: U(1) fermions + gauge bosons in 1D

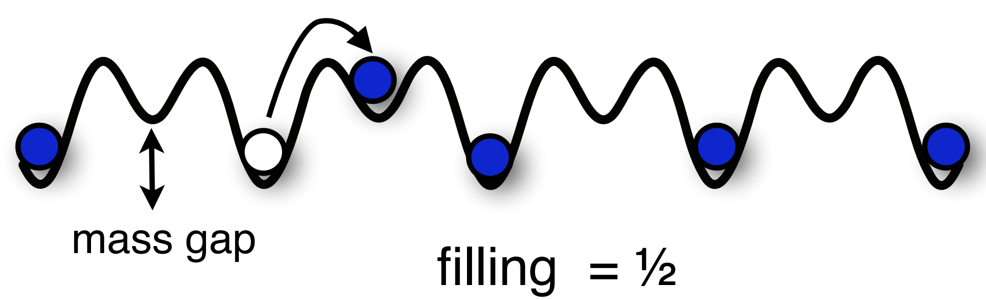
- **Hamiltonian:** staggered fermions in 1D coupled to quantum link spin S

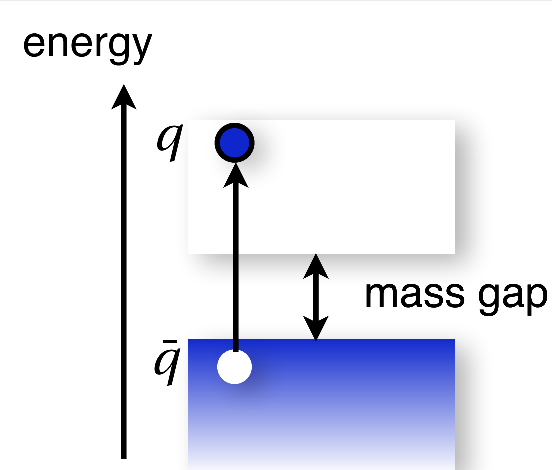
$$H = \frac{g^2}{2}\sum_x ({S^z}_{x,x+1})^2 - t\sum_x \left[\psi_x^\dagger S^+_{x,x+1}\psi_{x+1} + \mathrm{h.c.}\right] + m\sum_x (-1)^x \psi_x^\dagger \psi_x$$

electric flux — hopping — staggered fermions



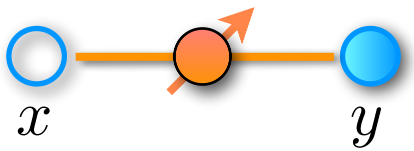
Staggered fermions:





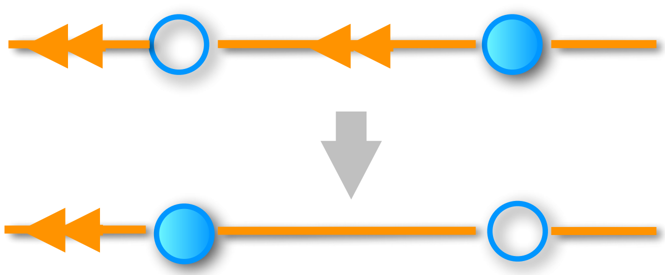
Implementing the Hopping Term

- **Abelian U(1)**



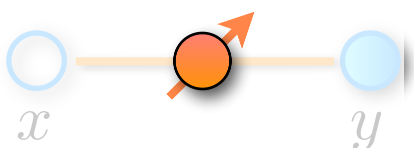
$$H = -t\psi_x^\dagger S_{xy}^+ \psi_y + \text{h.c.} + \dots$$

Dynamics:



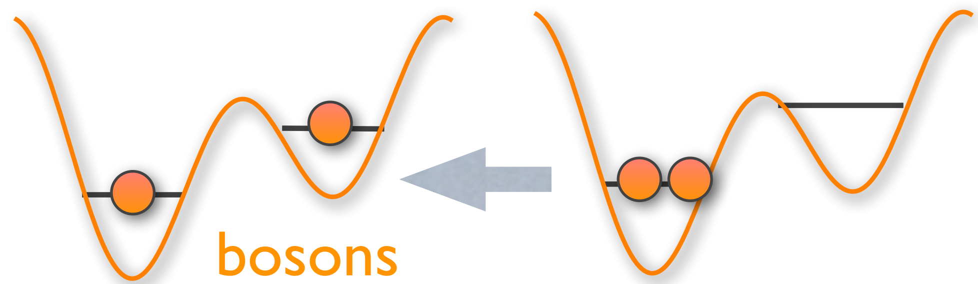
Implementing the Hopping Term

- **Abelian U(1)**

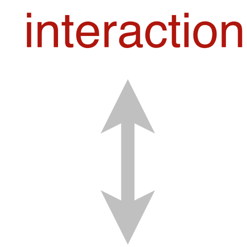


$$H = -t\psi_x^\dagger S_{xy}^+ \psi_y + \mathrm{h.c.} + \dots$$

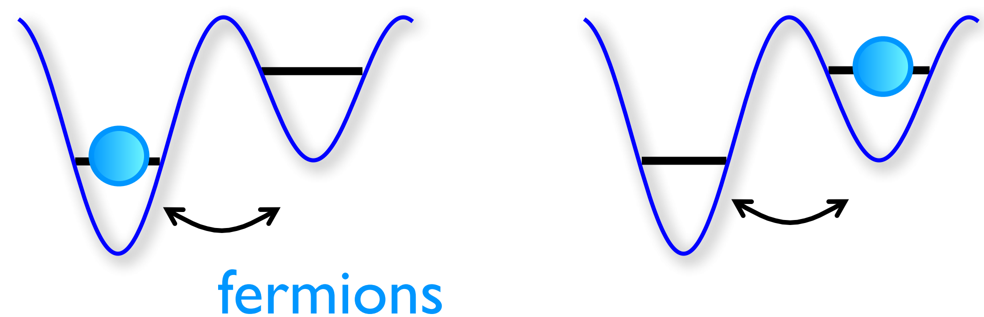
Spin as Schwinger Bosons: S=1



$$S_{x,y}^+ = b_L^\dagger b_R$$



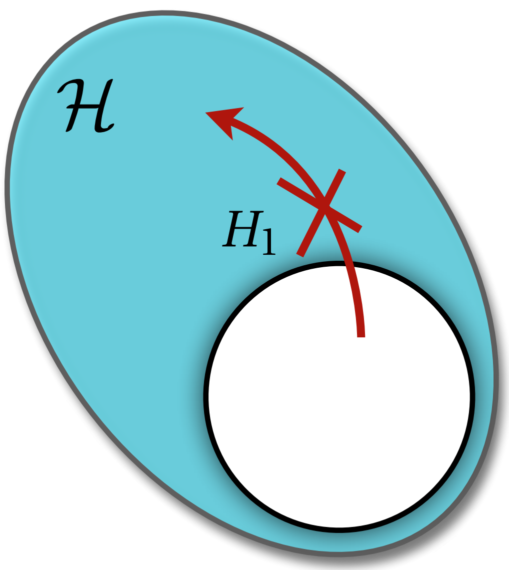
correlated hopping



$$H = -\frac{t_B t_F}{U_{BF}}\, \psi_x^\dagger \underbrace{b_L b_R^\dagger}_{\equiv U_{xy}} \psi_y + \mathrm{h.c.}$$

Implementing "Gauss Constraint"

- **Lattice Gauge Theory:** gauge symmetry fundamental
- **Implementation:** gauge symmetry approximate → protect against errors

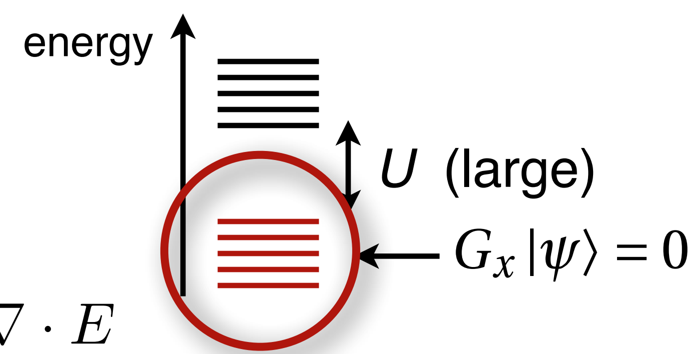


Strategies for Microscopic Implementation

1. Energy Constraints (as in cond mat)

$$H_{\text{micro}} = U \sum_x G_x^2 + \ldots$$

$$G_x \sim \rho - \nabla \cdot E$$

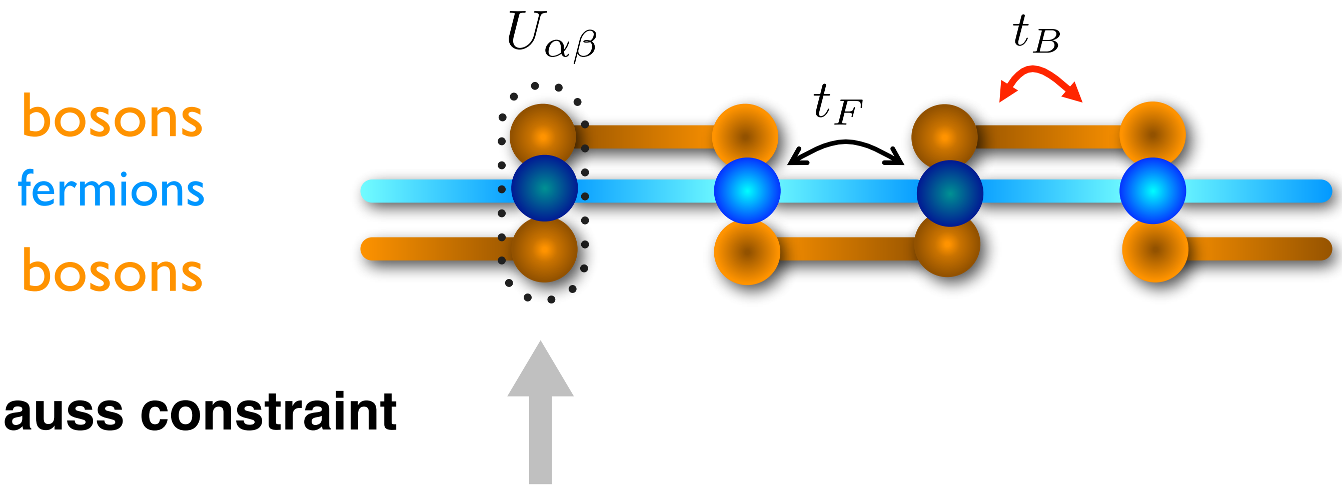


✓interaction

✓emergent lattice gauge theory (low energy)

Implementating Gauss Constraints

- **Bose-Fermi mixtures in superlattices**



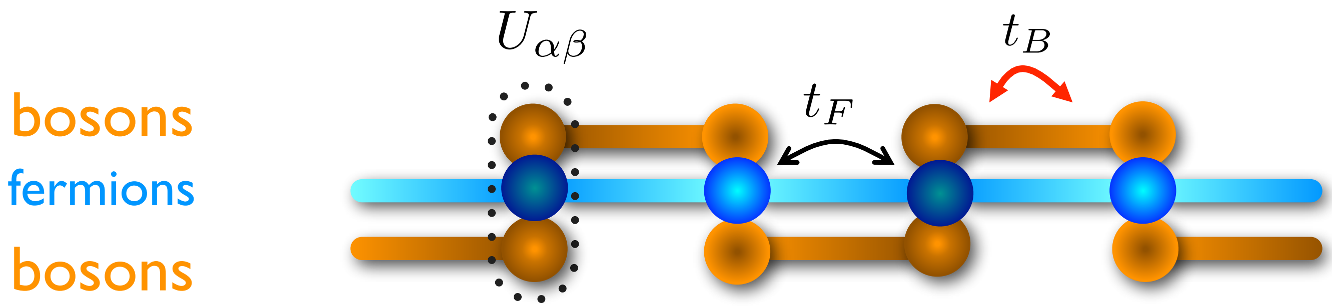
- **Gauss constraint**

$$\tilde{G}_x = n_x^F + n_x^1 + n_x^2 - 2S + \frac{1}{2}\left[(-1)^x - 1\right]$$

~ total number of atoms on site x fixed: "super-Mott insulator"

Implementating Gauss Constraints

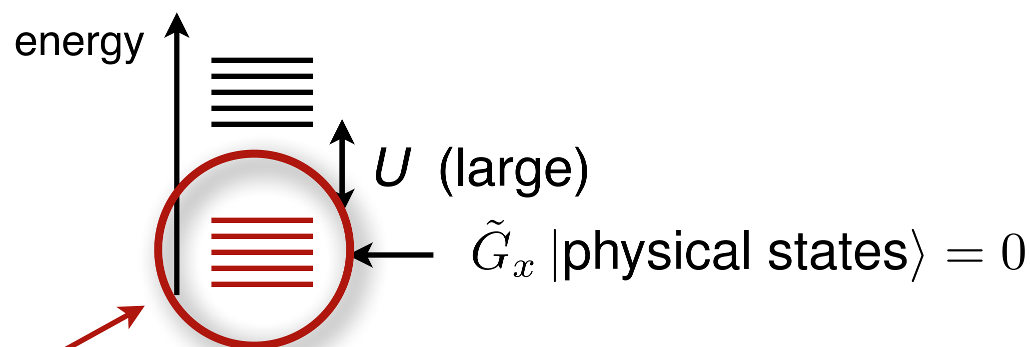
- **Bose-Fermi mixtures in superlattices**



- **enforcing the Gauss Law as an *energy constraint***

$$H_{\text{microscopic}} = U \sum_x \tilde{G}_x^2 + \ldots$$

Bose + Fermi Hubbard model

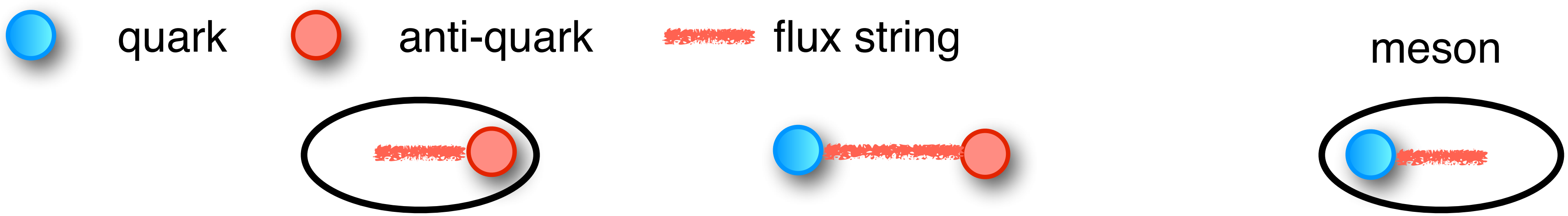


- emergent lattice gauge theory

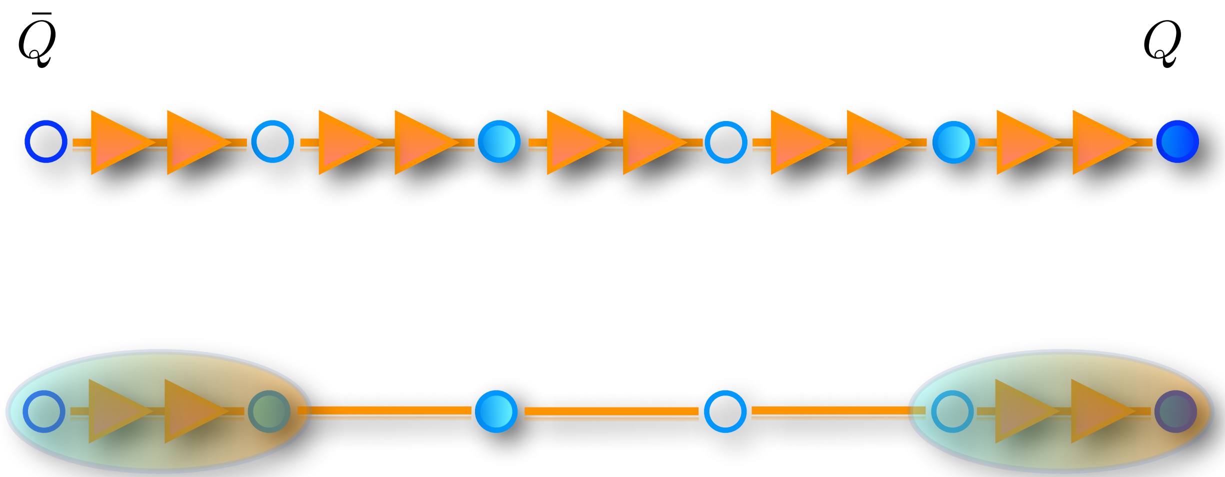
  - dynamics in physical subspace: analogous to t-J model

  - we have verified the reduction: microscopic to the quantum link model at the few- and many-body level

String breaking and confinement



Unbroken String



$$E_{\text{string}} - E_0 = \frac{g^2}{2}(L-1)$$

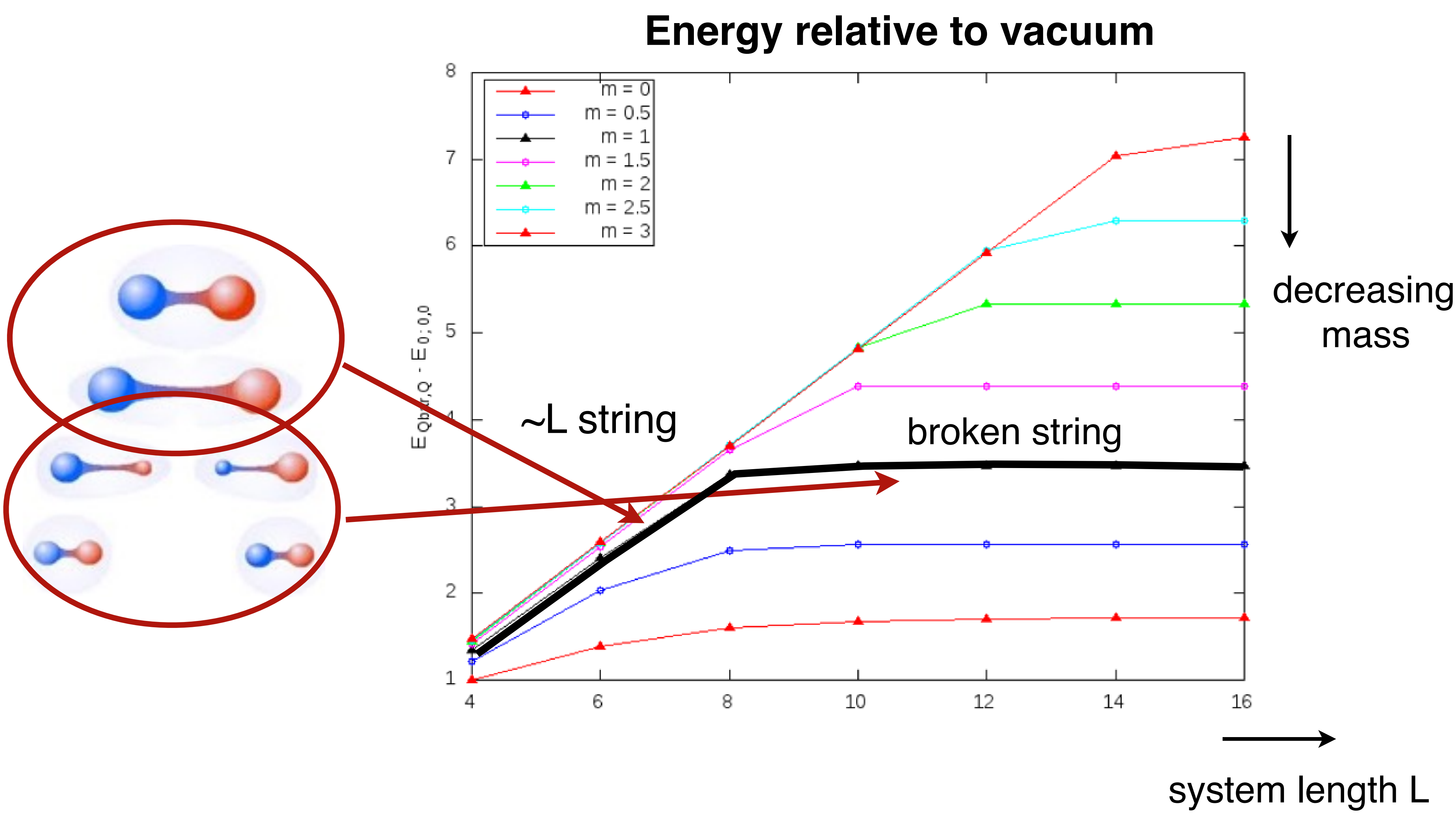
Mesons + vacuum

$$E_{\text{mesons}} - E_0 = 2\left(\frac{g^2}{2} + m\right)$$

Critical string length $L^{(c)} = 2 + 2m/g^2$

Rem.: string breaking 1D vs. 3D / non-Abelian

Spin-1: String Breaking



Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

doi:10.1038/nature18318


E. Martinez C. Muschik

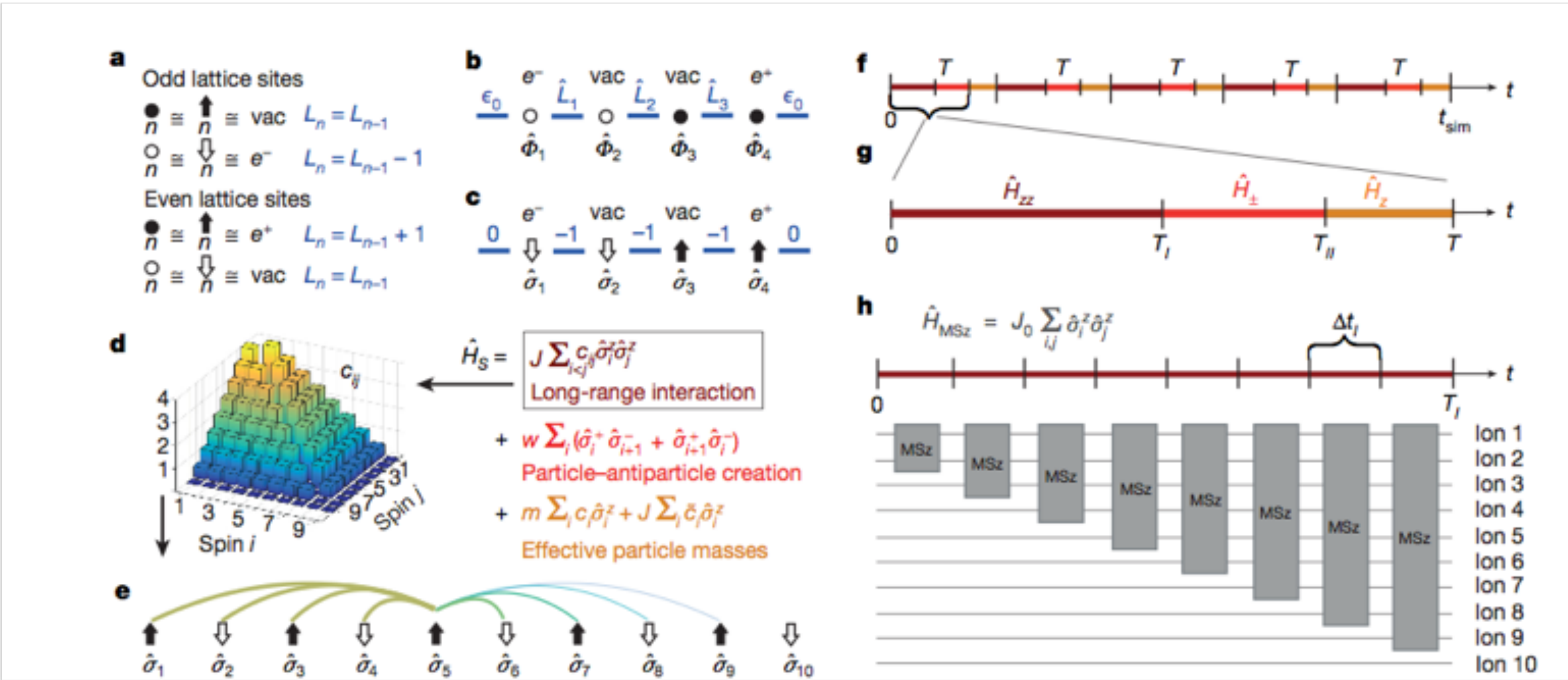


Figure 2 | Encoding Wilson's lattice gauge theories in digital quantum simulators. Matter fields, represented by one-component fermion fields

see talk by R. Blatt this Saturday